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Extensional Petri Net

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Abstract. Petri nets form a concurrent model for distributed and asynchronous systems. They are capable of modeling information flow in a closed system, but are generally not suitable for the study of compositionality. We address the issue of Petri net compositionality by introducing extensional Petri nets. In an extensional Petri net some places are external while others are internal. Every external place is labeled by a distinguished interface name. When composing two extensional Petri nets two places with a same label are coerced. An external place can be turned into an internal place by applying localization operator. The paper takes a look at bisimulation semantics and observational properties of the extensional Petri nets.

Keywords: Petri Net, Compositionality, Bisimulation

1. Introduction

Petri nets [Per81] form a model for concurrency and causality. They are best in describing information flows in closed isolated hardware systems. It is well-known that Petri net theory lacks of a good notion of compositionality. Without compositionality it is difficult to apply Petri nets to study the dynamic aspect of mobile and distributed computing systems. In modern computing everything is connected. Systems are widely distributed and highly concurrent. A system is bound to interact with environments in one way or another. The demand for compostionality is stronger than ever. In real applications Petri nets tend to be very large and very complex. The design of huge Petri nets is managed by designing smaller components, and then sticking the components through interfaces to form larger Petri nets. A notion of composition is already there in this design methodology. Accordingly the analysis of the property of the large Petri nets can be carried out by analyzing the local property of the component Petri nets and then obtain the global property by additional analysis. This is an obvious effective way to control the complexity of the analysis of large systems. In summary for both practical and theoretical purposes Petri nets are best seen as open systems [Hao97] that can be built up and analyzed in a structural manner. In this paper we formalize the place-transition nets so that they are open systems.

Interactions always occur at interfaces. In process calculi the interfaces are channels. Two processes may communicate when they are connected to the two ends of a channel. To compose Petri nets we need

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to pin down the interfaces. In literature there are different ways to define composition [Aal05, BDK01, BDK2001, BuG95, BuG09, CWY04, DKK04, DNM88, FuL10, Fuy16, GHH08, Gol90, Nie87, YCX07]. There is a synchronization approach that allows transitions from different nets to be performed as a single action. There is a shared memory approach that identifies places of different nets. There is a research line that combines the Petri net theory and the process algebraic theory, aiming at a unified framework of both. There is also proposal that defines composition by categorical pushout.

Systems are composed because they have to interact. In literature there are two ways to define interactions between nets. In transition based cooperations two or more transitions from different component nets are synchronised at correlated transition labels. Synchronization is achieved in a most direct fashion. This is apparently very much influenced by the interaction mechanism of CCS [Mil89]. In place based scenario nets are composed by coercing certain places from different nets. There is real information flow between component nets, achieving the effect of message sharing mechanism. There are several variations in this setup. The open Petri nets introduced by Baldan et. al. [BBG09, BCE01, BCE05, BCE07, BCE08] are different from the variant proposed by [LJZ13]. In such nets some places with labels, called *open* or *external*, are considered as interfaces with environments. Among the external places, some are input places, others are output places. An external place can be both an input and an output one at the same time. This could be distinguished by introducing dangling arcs attached to the places. In addition to external places, there are synchronization transitions. When two nets are composed, the connections of transitions to their pre-set and post-set should be preserved. New connections cannot be added. In the larger net, a new arc may be attached to a place only if the corresponding place of the subnet has a dangling arc in the same direction. Dangling arcs may be removed, but cannot be added in the larger net. Baldan et. al studied the denotational semantics of such open nets. Finally there are of course hybrid models that have both transition based and place based features at the expanse of complicated syntax and semantics.

The configuration of a Petri net is fixed, whereas that of a process, say a CCS process, is dynamic. That compares well to the difference between hardware and software. The advantage of fixed configurations is that they are subject to investigations using combinatorial (graph theoretical) tools and linear algebra. On the other hand the success of process algebra is that they offer an observational theory for interaction, without which one cannot talk about any theory for composition. Our motivation in this paper is to redefine Petri nets in such a way that compositionality is a born feature rather than an enforced property. In other words we are interested in the following question: What would Petri nets possibly look like had Petri anticipated the importance of compositionality? He surely had not seen any process calculus back in 1962. At a theoretical level we hope to enrich the nice combinatorial and algebraic theory of Petri nets by an observational theory of nets. The nets we propose in this paper, called extensional Petri nets, can be seen as a simplification of the open Petri nets. In our approach only external places act as interfaces. We think of a place as a pool for tokens or a shared memory. Accordingly we do not distinguish between input places and output places. In our way of thinking transitions in Petri nets are internal actions. There is no way to synchronize transitions.

Observational equivalence on labeled nets has been studied extensively [BDH92, KoE99, KEB94, MeM90, NPS95, PrW98, Win87] using bisimulation semantics [Bek84, Hoa78, Kin97, Mil89, MPW92]. Often a label is attached to a transition so that the standard labeled transition semantics can be applied. The methodology we will use in this paper is different in a couple of accounts. Firstly since the transitions are all internal actions the labeled bisimulation semantics is out of place. The appropriate semantics ought to be the barbed semantics [MiS92]. Secondly we pay particular attention to divergence. This makes sense in the setting of Petri net model where all transitions are seen as internal actions [Fuy15]. The congruence relation we will introduce for the extensional Petri nets is an instance of the model independent equality of interactive objects studied in [Fuy16]. The model independence is important in our opinion because we do not want the equality of nets to have anything to do with a particular process calculus or a particular process equivalence.

The purpose of this paper is to present the very basics of the extensional Petri net theory. We will be content with the definition of the model and the bisimulation properties of the extensional Petri nets. We will point out that the well-known properties about the Petri nets, like coverability, reachability and others, have quite different and richer interpretations in the extensional model.

Section 2 defines extensional Petri nets and introduces two operations on these nets. Section 3 defines extensional equality on the extensional Petri nets and provides an alternative characterization. Section 4 explains why the standard Petri net properties need be modified for the extensional Petri nets. Section 5 discusses the relationship to previous works and points out a few research issues.

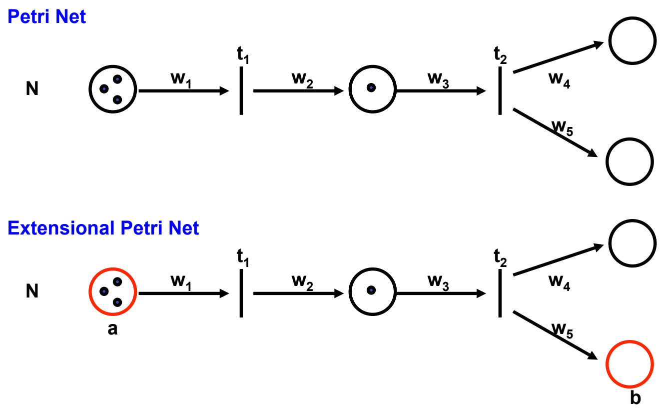


Fig. 1. A Petri net and an extensional net

2. Extensional Petri Net

Consider a system consisting of a laptop connected to both a printer and a projector. It does not make sense to think that any computation step executed within the laptop can be coupled with some computation step within the projector. Thus it is pointless to label any computation of a physical device because it cannot be seen by any other devices. The correct modelling of the situation is that the laptop sends some information to some place, and the projector picks up the information from that place. The interaction between the two devices must happen through the interface, which is say a physical connection. Similarly the laptop may deliver some information to some other place, from which the printer can fetch the message. In this case the connection may be wireless. So in a broader picture the system contains a subsystem modelling the wireless communication. The subsystem has a place to which the laptop sends the information and another place accessible by the printer. This simple example serves to explain our design principle: Transitions within a Petri net are internal actions not observable by any environments; the composition interfaces between Petri nets are places. When two nets are composed, some places in one net are merged with some places of the other net. To specify which is merged with which, we assign labels to places. In the following definition $\mathcal{N}$ is the set of interface names, $\mathbb{Z}$ is the set of integers, and $\mathbb{N}$ is the set of nonnegative integers.

Definition 2.1. An *extensional Petri net*, or *extensional net*, is a 5-tuple $\mathfrak{N} = \langle P, T, F, W, I\rangle$ where

1. P is the finite set of *places*, ranged over by s and its decorated versions;
2. T is the finite set of *transitions*, ranged over by t and its decorated versions, such that $P \cap T = \emptyset$;
3. $F \subseteq P{\times}T \cup T{\times}P$ is the *flow relation*;
4. $W : F \to \mathbb{N} \setminus \{0\}$ is the *weight function*;
5. $I : P \to \mathcal{N}$ is the *interface function*, which is a partial function injective on its domain of definition.

A *marking* of $\mathfrak{N}$ is a function $M : P \to \mathbb{N}$. If $M(s) = k$ we say that the place s contains k *tokens*. An *empty* marking is one in which no places contain any tokens.

Given an extensional net $\mathfrak{N}$, we write $P_{\mathfrak{N}}$, $T_{\mathfrak{N}}$, $F_{\mathfrak{N}}$, $W_{\mathfrak{N}}$, $I_{\mathfrak{N}}$ for the components of the net. We write $(\mathfrak{N}, M)$ to indicate that M is a marking of the extensional net $\mathfrak{N}$. Most of the time we simply write M, leaving the underlying extensional net implicit. In Fig. 1 there is an example of extensional net with marking.

Since every P/T-net can be transformed to a net with unlimited capacities without affecting its behavior, we will assume that every extensional net has unlimited capacity for all its places. A place of an extensional net is *external* if it is labeled by an interface name; it is *internal* if it is unlabeled. By definition no two external nodes have the same label. We shall write P^e for the set of external places in P and P^i for the set of internal places.

We think of a net as a physical device, and tokens as representing information. The transitions describe how information flows within the device. The following standard terminologies help to define the dynamics of the extensional nets.

Definition 2.2. Let $\mathfrak{N}$ be an extensional net and M be a marking of $\mathfrak{N}$.

1. For $x \in P_{\mathfrak{N}} \cup T_{\mathfrak{N}}$, ${}^{\bullet}x = \{y \mid yF_{\mathfrak{N}}x\}$ is the preset of x; $x^{\bullet} = \{y \mid xF_{\mathfrak{N}}y\}$ is the postset of x.
2. A transition $t \in T_{\mathfrak{N}}$ is M-*enabled*, notation $M[t\rangle$, if $\forall s \in {}^{\bullet}t.M(s) \geq W_{\mathfrak{N}}(s,t)$.
3. An M-enabled transition t may produce a *follower marking* M' defined as follows.

$$M'(s) \stackrel{\text{def}}{=} \begin{cases} M(s) - W_{\mathfrak{N}}(s,t), & \text{if } s \in {}^{\bullet}t \setminus t^{\bullet}, \\ M(s) + W_{\mathfrak{N}}(t,s), & \text{if } s \in t^{\bullet} \setminus {}^{\bullet}t, \\ M(s) - W_{\mathfrak{N}}(s,t) + W_{\mathfrak{N}}(t,s), & \text{if } s \in {}^{\bullet}t \cap t^{\bullet}, \\ M(s), & \text{otherwise.} \end{cases}$$

We write $M[t\rangle M'$ to indicate that M evolves into M' by *firing* t.

When studying observational behaviours of extensional nets, it is unnecessary to spell out which transition is fired because transitions are internal actions. We borrow the notation from process algebra by writing $M \xrightarrow{\tau} M'$ if $M[t\rangle M'$ for some transition t. The reflexive and transitive closure of $\xrightarrow{\tau}$ is denoted by $\Longrightarrow$.

An extensional net is able to interact with its environment through its external places. Two nets interact by sharing some external places. Extensional nets are thus composed by merging places with identical labels. We shall always assume that two extensional nets $\mathfrak{N}_1, \mathfrak{N}_2$ have disjoint sets of places ($P_{\mathfrak{N}_1} \cap P_{\mathfrak{N}_2} = \emptyset$) and disjoint sets of transitions ($T_{\mathfrak{N}_1} \cap T_{\mathfrak{N}_2} = \emptyset$). The following notations are used.

$$P^e_{\mathfrak{N}_1 \setminus \mathfrak{N}_2} = \{s \mid s \in P_{\mathfrak{N}_1} \ \wedge \ \neg\exists s' \in P_{\mathfrak{N}_2}.I_{\mathfrak{N}_1}(s) = I_{\mathfrak{N}_2}(s')\},$$
$$P^e_{\mathfrak{N}_1 \cap \mathfrak{N}_2} = \{\langle s_1, s_2\rangle \mid s_1 \in (P_{\mathfrak{N}_1})^e \ \wedge \ s_2 \in (P_{\mathfrak{N}_2})^e \ \wedge \ I_{\mathfrak{N}_1}(s_1) = I_{\mathfrak{N}_2}(s_2)\}.$$

In other words $P_{\mathfrak{N}_1 \setminus \mathfrak{N}_2}$ is the set of all the internal places of $\mathfrak{N}_1$ plus all the external places of $\mathfrak{N}_1$ that do not connect to $\mathfrak{N}_2$, while $P_{\mathfrak{N}_1 \cap \mathfrak{N}_2}$ is the set of pairs of external places of same label. Every pair of places in $P_{\mathfrak{N}_1 \cap \mathfrak{N}_2}$ are coerced in the composition. The following definition formalises this intuition.

Definition 2.3. The *composition* of two extensional nets $\mathfrak{N}_1, \mathfrak{N}_2$, denoted by $\mathfrak{N}_1 \mid \mathfrak{N}_2$, is the extensional net $\mathfrak{N}$ defined as follows:

- $P_{\mathfrak{N}} = P^i_{\mathfrak{N}} \cup P^e_{\mathfrak{N}}$, where

$$P^i_{\mathfrak{N}} = P^i_{\mathfrak{N}_1} \cup P^i_{\mathfrak{N}_2},$$
$$P^e_{\mathfrak{N}} = P^e_{\mathfrak{N}_1 \setminus \mathfrak{N}_2} \cup P^e_{\mathfrak{N}_2 \setminus \mathfrak{N}_1} \cup P_{\mathfrak{N}_1 \cap \mathfrak{N}_2}.$$

- $T_{\mathfrak{N}} = T_{\mathfrak{N}_1} \cup T_{\mathfrak{N}_2}$.
- $F_{\mathfrak{N}}$ is the following relation

$$\begin{aligned} & F_{\mathfrak{N}_1} \restriction P_{\mathfrak{N}_1 \setminus \mathfrak{N}_2} \cup F_{\mathfrak{N}_2} \restriction P_{\mathfrak{N}_2 \setminus \mathfrak{N}_1} \\ & \cup \{\langle\langle s_1, s_2\rangle, t\rangle \mid \langle s_1, t\rangle \in F_{\mathfrak{N}_1}, \langle s_1, s_2\rangle \in P_{\mathfrak{N}_1 \cap \mathfrak{N}_2}\} \\ & \cup \{\langle t, \langle s_1, s_2\rangle\rangle \mid \langle t, s_1\rangle \in F_{\mathfrak{N}_1}, \langle s_1, s_2\rangle \in P_{\mathfrak{N}_1 \cap \mathfrak{N}_2}\} \\ & \cup \{\langle\langle s_1, s_2\rangle, t\rangle \mid \langle s_2, t\rangle \in F_{\mathfrak{N}_2}, \langle s_1, s_2\rangle \in P_{\mathfrak{N}_1 \cap \mathfrak{N}_2}\} \\ & \cup \{\langle t, \langle s_1, s_2\rangle\rangle \mid \langle t, s_2\rangle \in F_{\mathfrak{N}_2}, \langle s_1, s_2\rangle \in P_{\mathfrak{N}_1 \cap \mathfrak{N}_2}\}. \end{aligned}$$

- $W_{\mathfrak{N}}$ is the function defined by

$$W_{\mathfrak{N}}(\langle s, t\rangle) = \begin{cases} W_{\mathfrak{N}_1}(\langle s, t\rangle), & \text{if } s \in P_{\mathfrak{N}_1 \setminus \mathfrak{N}_2}, \\ W_{\mathfrak{N}_2}(\langle s, t\rangle), & \text{if } s \in P_{\mathfrak{N}_2 \setminus \mathfrak{N}_1}, \\ W_{\mathfrak{N}_1}(\langle s_1, t\rangle), & \text{if } s = \langle s_1, s_2\rangle \ \wedge \ t \in T_{\mathfrak{N}_1}, \\ W_{\mathfrak{N}_2}(\langle s_2, t\rangle), & \text{if } s = \langle s_1, s_2\rangle \ \wedge \ t \in T_{\mathfrak{N}_2}. \end{cases}$$

  And dually for $W_{\mathfrak{N}}(\langle t, s\rangle)$.
- $I_{\mathfrak{N}}$ is the function defined as follows:

$$I_{\mathfrak{N}}(s) = \begin{cases} I_{\mathfrak{N}_1}(s), & \text{if } s \in P_{\mathfrak{N}_1 \setminus \mathfrak{N}_2}, \\ I_{\mathfrak{N}_2}(s), & \text{if } s \in P_{\mathfrak{N}_2 \setminus \mathfrak{N}_1}, \\ I_{\mathfrak{N}_1}(s_1), & \text{if } s = \langle s_1, s_2\rangle. \end{cases}$$

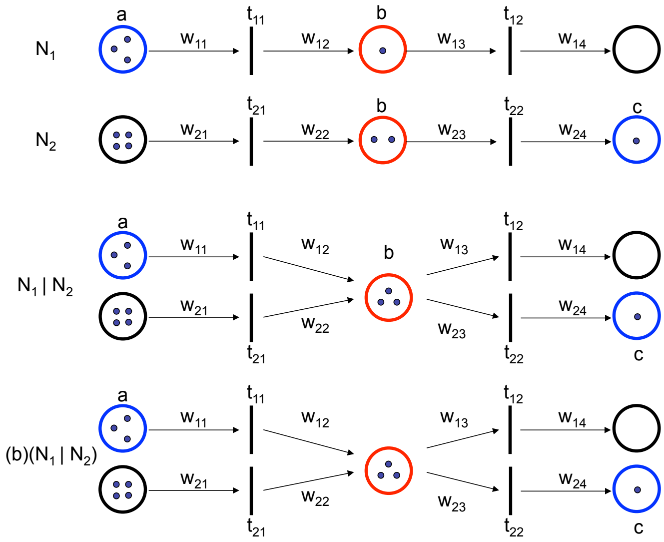


Fig. 2. Composition and localization

The composition of two markings M_1, M_2 of the nets $\mathfrak{N}_1, \mathfrak{N}_2$ respectively, notation $M_1 \,|\, M_2$, is the marking on $\mathfrak{N}_1 \,|\, \mathfrak{N}_2$ defined as follows:

$$(M_1 \,|\, M_2)(s) = \begin{cases} M_1(s), & \text{if } s \in P_{\mathfrak{N}_1 \setminus \mathfrak{N}_2}, \\ M_2(s), & \text{if } s \in P_{\mathfrak{N}_2 \setminus \mathfrak{N}_1}, \\ M_1(s_1) + M_2(s_2), & \text{if } s = \langle s_1, s_2 \rangle. \end{cases}$$

An extensional net is *locked* if none of its transitions is fireable, it is *deadlocked* if it is locked no matter how we change the number of tokens in its external places.

We also define a localization operator that removes the label of an external place and turns it into an internal place.

Definition 2.4. The *localization* of an extensional net $\mathfrak{N}$ at a name a, denoted by $(a)\mathfrak{N}$, is the extensional net obtained from $\mathfrak{N}$ by modifying the labeling function as follows:

$$I_{(a)\mathfrak{N}}(s) \stackrel{\text{def}}{=} \begin{cases} I_{\mathfrak{N}}(s), & \text{if } I_{\mathfrak{N}}(s) \neq a, \\ \uparrow, & \text{otherwise.} \end{cases}$$

A marking of $(a)\mathfrak{N}$ is the same thing as a marking of $\mathfrak{N}$, that is $(a)M = M$. Fig. 2 is a simple example of composition and localization. The localization operator turns the external place labeled b into an internal place. Once it has become internal, it no longer acts as an interface. Like in CCS both the composition operator and the localization operator can be applied unconditionally. If $\mathfrak{N}$ is an extensional net that does not have any external place labelled a, then $(a)\mathfrak{N}$ is semantically the same as $\mathfrak{N}$. The composition operator "|" is a must in any decent composition theory. Its prime role is to let interaction happen. The localization operator is important because it imposes control over interaction. Both operators are fundamental. The semantics of the composition operator is defined by the following standard structural rules:

$$\frac{M_1 \stackrel{\tau}{\longrightarrow} M_1'}{M_1 \,|\, M_2 \stackrel{\tau}{\longrightarrow} M_1' \,|\, M_2} \qquad \frac{M_2 \stackrel{\tau}{\longrightarrow} M_2'}{M_1 \,|\, M_2 \stackrel{\tau}{\longrightarrow} M_1 \,|\, M_2'}.$$

Back to the example we discussed in the beginning of the section. Let $\mathfrak{L}$, $\mathfrak{P}$ and $\mathfrak{D}$ be the extensional nets for the laptop, the projector and the printer respectively. Suppose both $\mathfrak{L}$ and $\mathfrak{P}$ have a place labeled p, and both $\mathfrak{L}$ and $\mathfrak{D}$ have a place labeled d. Then we can form the composite system $\mathfrak{L} \,|\, \mathfrak{P} \,|\, \mathfrak{D}$. The components $\mathfrak{L}$ and $\mathfrak{P}$ may interact through the place labeled p, and $\mathfrak{L}$ and $\mathfrak{D}$ may interact through the place labeled d. If we want to model the situation where the connection between the laptop and the projector is private, we obtain the system $(a)(\mathfrak{L} \,|\, \mathfrak{P}) \,|\, \mathfrak{D}$. In this system no other devices are allowed to connect to the projector.

3. Bisimulation for Extensional Nets

According to the viewpoint advocated in this paper the observables of an extensional Petri net are the external places since they are the interfaces through which the environments may test the net. Quantitively the status of an extensional net marking is determined by the number of tokens in the external places because these tokens can be consumed by the nets composed with it.

In concurrency theory the standard bisimulation equivalence is the weak bisimulation of Park and Milner [Par81, Mil89]. From the observational point of view there are two decidedly improvements of the weak bisimulation. The first is the barbed bisimulation of Milner and Sangiorgi [MiS92]. This is the first step towards a model independent characterization of process equality. The second is the branching bisimulation of van Glabbeek and Weijland [GlW89]. The branching approach draws a line between the internal transitions that change states and those that do not change states. It has been shown in a number of settings that the branching bisimilarity is more stable than the weak bisimilarity.

We define in this section an observational equivalence for the extensional nets. The particular equality we are interested in is the absolute equality studied in [Fuy16]. This is the equality for both computation models and interaction models. As an equality for computation it should be both bisimilar in the sense of van Glabbeek and Weijland [GlW89] and divergence sensitive in the sense of Priese [Pri78]. Hence the following.

Definition 3.1. A binary relation $\mathcal{R}$ on extensional net markings is a *bisimulation* if the following are valid whenever $M_1\mathcal{R}M_2$.

1. If $M_1 \xrightarrow{\tau} M_1'$, then either $M_2 \Longrightarrow M_2'$ for some M_2' such that $M_1\mathcal{R}M_2'$ and $M_1'\mathcal{R}M_2'$, or $M_2 \Longrightarrow M_2'' \xrightarrow{\tau} M_2'$ for some M_2'', M_2' such that $M_1\mathcal{R}M_2''$ and $M_1'\mathcal{R}M_2'$.
2. If $M_2 \xrightarrow{\tau} M_2'$, then either $M_1 \Longrightarrow M_1'$ for some M_1' such that $M_1'\mathcal{R}M_2$ and $M_1'\mathcal{R}M_2'$, or $M_1 \Longrightarrow M_1'' \xrightarrow{\tau} M_1'$ for some M_1'', M_1' such that $M_1''\mathcal{R}M_2$ and $M_1'\mathcal{R}M_2'$.

Definition 3.2. A binary relation $\mathcal{R}$ on extensional net markings is *codivergent* if the following are valid whenever $M_0\mathcal{R}M_0'$.

1. If $M_0 \xrightarrow{\tau} M_1 \xrightarrow{\tau} M_2 \xrightarrow{\tau} M_3 \xrightarrow{\tau} \ldots$ then $M_0' \xrightarrow{\tau} M''$ for some M'' such that $M_i\mathcal{R}M''$ for some $i > 0$.
2. If $M_0' \xrightarrow{\tau} M_1' \xrightarrow{\tau} M_2' \xrightarrow{\tau} M_3' \xrightarrow{\tau} \ldots$ then $M_0 \xrightarrow{\tau} M'$ for some M' such that $M'\mathcal{R}M_i'$ for some $i > 0$.

From the point of view of interaction the equality should be compositional and preserve the capacity to interact. The compositionality property is formalized next.

Definition 3.3. A binary relation $\mathcal{R}$ on extensional net markings is *extensional* if the following are valid.

1. $(M_1 \,|\, M_2)\,\mathcal{R}\,(M_1' \,|\, M_2')$ whenever $M_1\mathcal{R}M_1'$ and $M_2\mathcal{R}M_2'$.
2. For every interface name a, $(a)M\,\mathcal{R}\,(a)M'$ whenever $M\mathcal{R}M'$.

To introduce the next condition we need the important notion of observation. We will write C, D and their decorated versions for finite subsets of $\mathcal{N}$.

Definition 3.4. An *observation* is a function $O : C \to \mathbb{N}$ such that $O(c) \geq 0$ for every $c \in C$. The *observation* of a marking M of an extensional net $\mathfrak{N}$ is the function $O_M : I_{\mathfrak{N}}(P^e_{\mathfrak{N}}) \to \mathbb{N}$ such that $O_M(a) = M(s)$ for the $s \in P^e_{\mathfrak{N}}$ satisfying $I_{\mathfrak{N}}(s) = a$. The observation of a marking M is *positive* if $O_M(a) > 0$ for some $a \in I_{\mathfrak{N}}(P^e_{\mathfrak{N}})$.

Let $O : C \to \mathbb{N}$, $O' : D \to \mathbb{N}$ be observations with $C \subseteq D$. We write $O \leq O'$ if $O(c) \leq O'(c)$ for all $c \in C$. We write $O = O'$ if $O \leq O' \leq O$.

Definition 3.5. A marking M is *observable*, notation $M\Downarrow$, if $M \Longrightarrow M'$ for some M' with positive $O_{M'}$.

Equal markings should be either both observable or both unobservable.

Definition 3.6. A relation $\mathcal{R}$ on markings is *equipollent* if $M\Downarrow \Leftrightarrow M'\Downarrow$ whenever $M\mathcal{R}M'$.

We are now able to define the absolute equality [Fuy16] for the extensional Petri nets. In this paper we give it a different name for the reason that the communication mechanism of the model is based on shared memory rather than message-passing.

Definition 3.7. The *extensional equality* on the extensional net markings, denoted by $=$, is the largest reflexive, extensional, equipollent, codivergent bisimulation on the extensional net markings.

It is easy to see that in the presence of reflexivity the union of a set of extensional, equipollent, codivergent bisimulations is an extensional, equipollent, codivergent bisimulation. Hence the well definedness of the extensional equality. For more motivations and properties about the absolute equality the reader is referred to [Fuy16]. For the applications of the absolute equality in computation and process models, consult [FuL10, Fuy15, Fuy17a, Fuy17b]. The extensional equality of nets is yet another application of the absolute equality.

When reasoning about a relation we sometimes need to take its closure in one way or another. The *extensional closure* of a binary relation $\mathcal{R}$ on the extensional net markings, denoted by $\mathcal{R}^*$ is the least relation satisfying the following: (i) $\mathcal{R} \subseteq \mathcal{R}^*$; (ii) $(M \,|\, M', M'' \,|\, M''') \in \mathcal{R}^*$ whenever $M\mathcal{R}^*M''$ and $M'\mathcal{R}^*M'''$.

Proposition 3.1. The relation $=$ is both an equivalence and a congruence.

Proof. The relation $=$ is reflexive by definition. The extensionality, equipollence, codivergence and bisimulation are symmetric and transitive. So $=$ is an equivalence. To prove congruence, one notice that $M = M'$ implies $(a)M = M = M' = (a)M'$ for every interface name a. So all one has to prove is that $=^*$ is an extensional, equipollent, codivergent bisimulation. This is routine. □

Definition 3.8. A renaming is an injective function $\varsigma : C \to \mathcal{N}$ for some finite $C \subseteq \mathcal{N}$. A *displacement* is a function $\delta : C \to \mathbb{Z}$ for some finite $C \subseteq \mathcal{N}$.

The notation $M\varsigma$ denotes the extensional net marking obtained by renaming the labels of M according to ς. The marking $M\delta$ is the marking M modified by the effect δ exerted by an environment. It is defined as follows.

$$M\delta(s) \stackrel{\rm def}{=} \begin{cases} M(s) + \delta(a), & \text{if } s \in P^e_{\mathfrak{N}} \text{ and } I_{\mathfrak{N}}(s) = a \text{ and } \delta \text{ is defined on } a, \\ M(s), & \text{otherwise,} \end{cases} \tag{1}$$

The map $M\delta$ is not necessarily a legal marking. In the rest of the paper whenever we write $M\delta$ we always assume that it is a legitimate marking. When we say $M\delta = N\delta$ for all δ we mean that $M\delta = N\delta$ holds for all such δ that both $M\delta$ and $N\delta$ are legitimate.

Lemma 3.1. Suppose ς is a renaming function. Then $M = M'$ implies $M\varsigma = M'\varsigma$.

Proof. Let $\mathcal{R}_{M=M'}$ be the subset of $=$ that contains all the pairs (M_1, M_1') of $=$ such that $M \Longrightarrow M_1$ and $M' \Longrightarrow M_1'$. Let ς be a renaming from $\{a\}$ to $\mathcal{N}$. It is easy to show that $\{(M\varsigma, M'\varsigma) \mid (M, M') \in \mathcal{R}\} \cup =$ is the largest reflexive, extensional, equipollent, codivergent bisimulation. □

Before proving the next property about the equality, let's recall some standard terminologies and technical lemmas. We say that $M \xrightarrow{\tau} M'$ is a *(deterministic) computation* step, notation $M \to M'$, if $M' = M$; it is a *nondeterministic computation* step, notation $M \xrightarrow{\iota} M'$, if $M' \neq M$. The reflexive and transitive closure of $\to$ is denoted by $\to^*$. If $M = N$ then $M \xrightarrow{\iota} M'$ must be bisimulated by $N \xrightarrow{\tau} N_1 \xrightarrow{\tau} \ldots \xrightarrow{\tau} N_k \xrightarrow{\tau} N'$ such that $N \to N_1 \to \ldots \to N_k$, $N_k \xrightarrow{\iota} N'$ and $M' = N'$. In some sense this is the property that justifies the definition of bisimulation. It is related to the following two fundamental lemmas.

Lemma 3.2 (Computation Lemma). If $M_0 \xrightarrow{\tau} M_1 \xrightarrow{\tau} \ldots \xrightarrow{\tau} M_k = M_0$ then $M_0 \to M_1 \to \ldots \to M_k$.

Lemma 3.3 (Bisimulation Lemma). If $M \Longrightarrow M' = N$ and $N \Longrightarrow N' = M$ then $M = N$.

The proofs of the two lemmas can be found in [Fuy16].

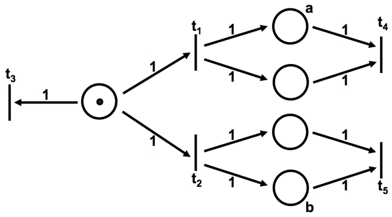
The extensional equality is often not easy to work with. The main difficulty is caused by the extensionality condition. We are going to give an important characterization of the extensional equality that bypasses the extensionality condition. The characterization makes use of the following lemma.

Lemma 3.4. If $M = N$ then the following hold: (i) $M \to^* M'$ for some M' such that $O_{M'} \geq O_N$ and $M'\delta = N\delta$ for all δ. (ii) $N \to^* N'$ for some N' such that $O_{N'} \geq O_M$ and $M\delta = N'\delta$ for all δ.

Proof. Suppose M, N are markings of the nets $\mathfrak{N}_1, \mathfrak{N}_2$ respectively and that $M = N$. Assume that $O_M \not\leq O_N$ and neither a nor b appears in $\mathfrak{N}_1 \,|\, \mathfrak{N}_2$. Let $\widetilde{c}$ be the set of all labels that appear in $\mathfrak{N}_1 \,|\, \mathfrak{N}_2$. Let $\mathfrak{N}$ be an extensional net with three transitions t_1, t_2, t_3, t_4, t_5 and let L be a marking on $\mathfrak{N}$ that assigns no tokens in any external places. When composed with $\mathfrak{N}_1$, the net $\mathfrak{N}$ can perform the following actions.

- It can fire t_1 whose precondition requires all the tokens in the external places of $\mathfrak{N}_1$. After the action the observation of the marking of $\mathfrak{N}_1$ is $M\delta$ and a token in the place labeled a is produced. Having fired t_1 the transition t_4 can be fired to deadlock $\mathfrak{N}$.
- It can fire t_2 that produces a token in the place labeled by b and empties all the external places of $\mathfrak{N}_1$. Afterwards it can fire t_5 to deadlock $\mathfrak{N}$.
- It can fire t_3 to deadlock $\mathfrak{N}$ and produces the same number of tokens as it consumes in all external places.

Additionally we require that the transitions t_1, t_2 block each other. A diagrammatic illustration of $\mathfrak{N}$ with marking L is given below, in which we have ignored part of the net that connects to $\mathfrak{N}_1$.



Suppose that $M \,|\, L \xrightarrow{\iota} M\delta \,|\, L'$ is caused by firing t_1. This is a nondeterministic computation because it disables the capacity to produce a token in the place labeled b. If N cannot reach to a marking N' such that $O_{N'} \geq O_M$ then $(\widetilde{c})(M\delta \,|\, L')$ is observable whereas no descendant of $(\widetilde{c})(N \,|\, L)$ is observable. This is a contradiction. We conclude that there must be some N' such that $O_{N'} \geq O_M$ and for some L',

$$N \,|\, L \to^* N' \,|\, L \xrightarrow{\iota} N'' \,|\, L'' = M\delta \,|\, L'.$$

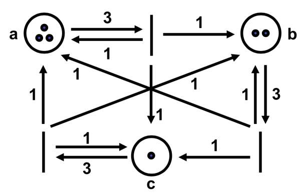
Now $N \,|\, L \xrightarrow{\iota} N \,|\, L_0 = N$, caused by the firing of t_3, must be bisimulated by $N' \,|\, L \to^* N_1' \,|\, L \xrightarrow{\iota} N_1' \,|\, L_0 = N_1'$ for some N_1', and $N' \,|\, L \xrightarrow{\iota} N' \,|\, L_0 = N'$, caused by the firing of t_3, must be bisimulated by $N \,|\, L \to^* N_1 \,|\, L \xrightarrow{\iota} N_1 \,|\, L_0 = N_1$ for some N_1. It follows that $N \Longrightarrow N_1 = N'$ and $N' \Longrightarrow N_1' = N$. Thus $N = N'$ by appealing to the Bisimulation Lemma. Also notice that L'' must be the same as L', otherwise a similar contradiction can be derived. It follows that $N'' = N'\delta$ and $N'\delta \,|\, L' = M\delta \,|\, L'$. With a marking L' the net $\mathfrak{N}$ can fire t_4 to deadlock itself we conclude from Bisimulation Lemma that $N'\delta = M\delta$. $\Box$

Having seen the Computation Lemma and Lemma 3.4, let's see a couple of examples.

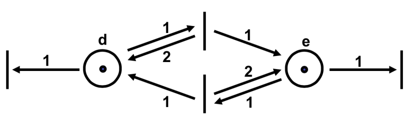
1. Define a three place extensional net, all the three places being external. Let's use the vector $\langle 1, 2, 3 \rangle$ to denote a marking. The net has three transitions that can be executed cyclically in the following manner.

   $$(1, 2, 3) \xrightarrow{\tau} (2, 3, 1) \xrightarrow{\tau} (3, 1, 2) \xrightarrow{\tau} (1, 2, 3).$$

   The transitions are defined in the following diagrammatic presentation of the extensional Petri net.

   By the Computation Lemma these three pairwise distinct markings are equal.
2. Suppose we have an extensional net with two external places. For either external place there is a transition that increases the number of tokens in both places and another transition that decreases the number of tokens by one. The diagrammatic presentation is given below.

Starting from any nonempty marking this extensional net can reach to any marking. It follows from the Computation Lemma that all nonempty markings of this extensional net are equal. The empty marking is not equal to any other marking.

The two examples also provide an intuition for Lemma 3.4.

Based on Lemma 3.4 we introduce another equality for the extensional net markings.

Definition 3.9. A codivergent bisimulation $\mathcal{R}$ on the extensional net markings is an *open bisimulation* if the following statements are valid whenever $M\mathcal{R}N$.

1. $M \Longrightarrow M'$ for some M' such that $O_{M'} \geq O_N$ and $M'\delta\mathcal{R}N\delta$ for all replacement δ.
2. $N \Longrightarrow N'$ for some N' such that $O_{N'} \geq O_M$ and $M\delta\mathcal{R}N'\delta$ for all replacement δ.

The *open bisimilarity* $\simeq$ is the largest open bisimulation.

This new equality is justified by the following coincidence result.

Theorem 3.1. The open bisimilarity coincides with the extensional equality.

Proof. The inclusion $=\subseteq\simeq$ follows from Lemma 3.4. In the other direction we need to prove that $\simeq$ is reflexive, equipollent and extensional. Reflexivity and equipollence are obvious from definition. For extensionality we only have to prove that $M \simeq N$ implies $M \,|\, L \simeq N \,|\, L$ and $L \,|\, M \simeq L \,|\, N$ for all extensional net marking L because that is equivalent to the extensionality condition. It is not difficult to verify that both $\{(M \,|\, L, N \,|\, L) \mid M \simeq N \text{ and } L \text{ is a marking}\}$ and $\{(L \,|\, M, L \,|\, N) \mid M \simeq N \text{ and } L \text{ is a marking}\}$ are open bisimulations. □

4. Observable Property for Extensional Nets

From the point of view of observation the number of tokens in any internal places is immaterial. If we see the nets from this perspective we should define Petri net properties that ignore all internal places. To explain the basic idea let's define coverability/reachability for the extensional nets.

Definition 4.1. Suppose $\mathfrak{N}$ is an extensional net, M is a marking of $\mathfrak{N}$ and O, O' are observations of $\mathfrak{N}$. We say that O' is *coverable* by M, respectively *reachable* from M, if $M \Longrightarrow M'$ for some marking M' such that $O_{M'} \geq O'$, respectively $O_{M'} = O'$. We say that O' is *coverable* by O, respectively *reachable* from O, if $M'' \Longrightarrow M'''$ for some markings M'', M''' such that $O_{M''} = O$ and $O_{M'''} \geq O'$, respectively $O_{M'''} = O'$.

It is well-known that both the coverability problem [KaM69, Rac78] and the reachability problem [SaT77, May81, Kos82, Lam92] for the Petri nets are decidable. The next two results show that the decidability proof of a problem in the extensional net theory is built upon the decidability proof of the corresponding problem in the Petri net theory. We assume that the reader is aware of the proof of the decidability of the coverability problem. The proof is not short, so it cannot be repeated here.

Theorem 4.1. Given observations O, O' of an extensional net $\mathfrak{N}$ and a marking M of $\mathfrak{N}$, it is decidable to check if O' is coverable by O respectively M.

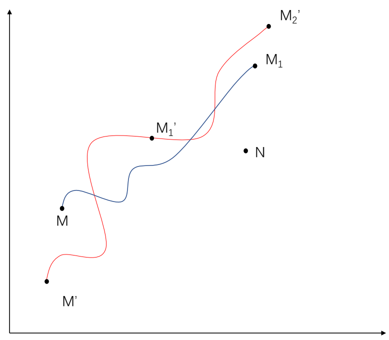
Proof. Using Rackoff's proof [Rac78] we argue that it is decidable to check if O' is coverable by O. Suppose $\mathfrak{N}$ has n places and e external places. We write a marking of the net as an n-tuple $\langle v_1, \ldots, v_n\rangle \in \mathbb{N}^n$ and an external marking as an e-tuple $\langle v_1, \ldots, v_e\rangle \in \mathbb{N}^e$. We identify O to $\langle u_1, \ldots, u_e\rangle$ and O' to $\langle w_1, \ldots, w_e\rangle$. The question is if there are $u_{e+1}, \ldots, u_n, u'_1, \ldots, u'_n \geq 0$ such that $\langle u_1, \ldots, u_n\rangle \Longrightarrow \langle u'_1, \ldots, u'_n\rangle$ and $\langle u'_1, \ldots, u'_e\rangle \geq \langle w_1, \ldots, w_e\rangle$. Suppose $\mathbf{z} = \langle z_1, \ldots, z_n\rangle \in \mathbb{Z}^n$. We say that $\mathbf{z}$ is i-bounded if $0 \leq z_j$ for all j such that $0 \leq j \leq i$, and that $\mathbf{z}$ is i-r-bounded, where $r > 0$, if $0 \leq z_j \leq r$ for all j such that $0 \leq j \leq i$. A sequence $\mathbf{z}_1, \mathbf{z}_2, \ldots, \mathbf{z}_s$ is i-bounded/i-r-bounded if each $\mathbf{z}_t$, where $1 \leq t \leq s$, is i-bounded/i-r-bounded. The sequence is i-covering if $\mathbf{z}_s(j) \geq u_j$ for all j such that $1 \leq j \leq i$. Rackoff proved that if there is an i-bounded and i-covering sequence, where $1 \leq i \leq n$, from $\langle u_1, \ldots, u_n\rangle$ then there is such a sequence whose length is bounded by $2^{2^{p(m,i)}}$, where $p(x,i)$ is a polynomial and m is the input size. Using this result it is easy to see that the coverability question is equivalent to asking if $\langle w_1, \ldots, w_e, 0, \ldots, 0\rangle$ can be covered by a path from $\langle v_1, \ldots, v_e, m2^{2^{p(m,e)}}, \ldots, m2^{2^{p(m,e)}}\rangle$, which is decidable. The argument for the other case is similar. □

Similarly we can derive the decidability of the reachability problem in the extensional setting. The reader is referred to the literature for the highly nontrivial decidability proof.

Theorem 4.2. Given two observations O, O' of an extensional net $\mathfrak{N}$ and a marking M of $\mathfrak{N}$, it is decidable to check if O' can be reached from O respectively M.

Proof. We can prove the theorem using the well-known KLMST algorithm [SaT77, May81, Kos82, Lam92]. A simple argument is also possible in the case of checking O reaching O'. It is decidable to check if $\langle u_1, \ldots, u_e\rangle$ can reach to $\langle w_1, \ldots, w_e\rangle$. If the answer is yes, we apply a result in [LeS15] stating that there is a bound, computable from the input net, on the length of the minimal path from $\langle u_1, \ldots, u_e\rangle$ to $\langle w_1, \ldots, w_e\rangle$. Using this bound we can easily compute $\langle u_{e+1}, \ldots, u_n\rangle$ and $\langle w_{e+1}, \ldots, w_n\rangle$ such that $\langle w_1, \ldots, w_e, w_{e+1}, \ldots, w_n\rangle$ is reachable from $\langle u_1, \ldots, u_e, u_{e+1}, \ldots, u_n\rangle$. □

Further questions about coverability and reachability can be asked. Suppose we know that O_N can be covered by O_M and that $M' = M$. Is it true that O_N can be covered by $O_{M'}$? The answer is positive. This is because $M \Longrightarrow M_1$ for some M_1 such that $O_{M_1} \geq O_N$. Since $M' = M$ there must be some M_1' such that $M' \Longrightarrow M_1' = M_1$. By Lemma 3.4 some M_2' exists such that $M_1' \to^* M_2'$ and $O_{M_2'} \geq O_{M_1}$. Thus $O_{M_2'} \geq O_N$.



Now consider a similar scenario. Suppose that O_N can be covered by O_M and that $N' = N$. Is it the case that N' is coverable by M? By assumption $M \Longrightarrow M'$ for some marking M' such that $O_{M'} \geq O_N$. By Lemma 3.4 there is some N_1 such that $N \Longrightarrow N_1$ and $O_{N_1} \geq O_{N'}$. The problem is that the transition sequence from N to N_1 is not necessarily a legal transition sequence from M' due to the presence of internal places. Reachability satisfies even less algebraic property. Even if O_N is reachable from O_M and $M' = M$, we know nothing about the reachability of O_N from $O_{M'}$. For example M may have the following transition sequence $(0,0) \to (2,2) \to (4,4) \to \ldots \to (2i,2i) \to \ldots$ whereas M' may have the following transition sequence $(1,1) \to (3,3) \to (5,5) \to \ldots \to (2i+1,2i+1) \to \ldots$. It can be the case that M reaches to N if and only if M' never reaches to N. Both coverability and reachability enjoy better algebraic property when confined to those extensional nets whose internal markings are bounded. This is a reasonable restriction if nets are regarded as physical devices.

5. Remark

We have demonstrated that by simply assigning interface names to places that are meant to be interfaces we obtain a variant of Petri net model that enjoys nice compositional property as well as a smooth observational theory. The extensional theory offers a new dimension for future investigations. The algebraic theory of the extensional nets should be systematically studied. We expect that a property defined in terms of external places is algebraic in the sense that it is preserved by the extensional equality. Many decidability results and algorithms in Petri net theory should be reexamined in this new framework. As we have seen the problems are often generalized when casting in the setting of the extensional nets. Hopefully studies on the generalized problems will shed new light on these problems. This paper is only a starting point for a general compositional theory of Petri nets.

The observational theory of nets studied in this paper is different from any observational theory proposed in literature. We have applied the model independent approach of the theory of interaction to Petri nets. Apart from obtaining a consistent theory the model independent approach allows one to study further issues like completeness and expressiveness [Fuy16] of Petri net model. What is for example the relationship between the extensional Petri net model and the value-passing calculus [Hoa78, Mil89, Fuy13]? How about the incompatibility result between the extensional Petri net model and the (higher order) π-calculus [MPW92, FuZ15, Fuy17a]? What is a universal extensional Petri net [Fuy17b]? The observation theory of this paper has laid down the fundamental framework for answering all these interesting questions.

As is pointed out in the introduction there is now a large literature on the compositionality of Petri net. Compared to the most proposed models the extensional Petri model is quite light-weighted. Our goal is not to pursue an integrated model that introduces a whole range of process operators in order to have a better match to a process calculus. Net property analysis is hard in such a complicated model. Our model also differs greatly from the models that admit labelling on transitions and synchronisation of transitions. It is our opinion that a Petri net transition is different from a transition in a process calculus. In the latter model an input action or an output action is part of an interaction, whereas in the former model a transition is a complete action. The synchronisation of two transitions in two components of a Petri net can only be implemented through interfaces. Our approach to the compositionality of Petri net is not new. Assigning labels to places is a feature of a number of models proposed in literature. Most of the time it comes with other features that complicate models considerably. The motivation of this work is to solve the compositionality problem by introducing as few operators as possible so that the new model looks like Petri net model as much as possible. Our understanding of the issue is that the minimal set of operators necessary for a compositional theory only consists of the concurrent composition operator and the localization operator. The extensional Petri net model formalizes the view that a net is composed from individual transitions. So a structural theory is possible with the extensional Petri nets. Another advantage of our minimal model is that it inherits most of the Petri net properties from the classical theory. The combinatorial approach and the algebraic approach work just as well in the extensional scenario. The interplay between the net properties and the observational properties is an avenue for further study.

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