# Passing names with priority guards

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We introduces **priority guards** into the $\pi$-calculus whose communication may pass names along channels.. Priority guards are proposed by Phillips as an extension with priority as CPG (CCS with priority guards)[Phi08]. A priority guards is a set of names that prior to the associated name. The name passing allows recursively guarded names in priority guards. We give a simple interpretation for those nested priority guards. By the dynamic feature of name passing, it is shown that our extended calculus is able to deal with 'local' and 'dynamic' priorities in process calculus.

## Names with priority guards

A priority guard for CCS is a set of name as proposed by Phillips. Let $A$ be a set of name, then $A : a$ is a guarded name. A prefix with a guarded name is written as $A : a.P$. The intuitive meaning of the prefix is that $A : a.P$ becomes $P$ with a communication over $a$ only when no communication over $A$ is possible. For example, $\{a\} : b.P + a.Q|\overline{b} + \overline{a}$ has only one $\tau$ transition over name $a$, while a communication over $b$ is not possible since $b$ is guarded by $\{a\}$ and a communication over $a$ is possible.

$$\{a\}.b.P + a.Q|\overline{b} + \overline{a} \xrightarrow{\tau} Q|0$$

Priority does not unconditionally disable a communication over $b$. If $\overline{a}$ is not available, communication over $b$ is still possible.

$$\{a\}.b.P + a.Q|\overline{b} \xrightarrow{\tau} P|0$$

Phillips proposes an extension of CCS as CPG with the local priority. Similar to CCS, CPG has a scope defined by the restriction operator. Therefore, it is possible to limit the scope of the validity of priority. Camilleri and Winskel also propose a communicating calculus with the local priority by introducing the priority choice $P{+\!\rangle}Q$ where $P$ is prior to $Q$ in communications if both $P$ and $Q$ are ready to communicate. The difference between CPG and ${+\!\rangle}$ is ${+\!\rangle}$ does not allow mixed choice of inputs and outputs, whereas CPG does. Phillips proposes an operational semantics for CPG, then show a strong bisimulation theory. And it also investigate the expressiveness of CPG with the result that CPG cannot be encoded into the $\pi$ calculus.

## A name passing with priority guards

We try to extend the idea to a name passing calculus such as $\pi$ calculus. Since every name may have a priority guard, a name in a guard may have a priority guard. Following the standard name passing, a communication over $x$ carries a name with a priority guard.

$$\overline{x}\langle\{a\} : z\rangle|x(y).\{y\} : \overline{v}\langle w\rangle \xrightarrow{\tau} 0|\{\{a\} : z\} : \overline{v}\langle w\rangle$$

The nested priority guard $\{\{a\} : z\}$ defines if $\overline{v}$ is enabled or not. Following the priority meanings, when communications over $z$ is available, $\overline{v}$ is disabled. However, when communication over $a$ is available, $z$ is disabled. Therefore, when communication over $a$ is available, $\overline{v}$ is enabled. This means that $\overline{v}$ is disabled when $a$ is not available and $z$ is available. We write this disabling condition as $\neg a \wedge z$. We call this form a *name logic formula*. For a (nested) priority guard $G$, we write $[\![G]\!]$ for the corresponding name logic formula. The translation is recursively defined as follows.

$$[\![G]\!] = \begin{cases} \neg[\![G']\!] \wedge a & \text{if } G = G' : a \\ \vee_i [\![G_i]\!] & \text{if } G = \{G_1, \ldots, G_n\} \end{cases}$$

## A calculus with priority guards

Prefix $\pi$ is defined as follows.

$$\pi \quad ::= \quad G : \overline{x}\langle G' : y\rangle \mid G : x(G' : y) \mid G : \tau \mid [G : x = G' : y]\pi$$

If $G$ is empty, we write $a$ for $\emptyset : a$. When all priority guards are empty, the calculus is the $\pi$ calculus. Processes are given in the same way as the version of the $\pi$ calculus [SW01].

$$\begin{array}{rcl} P & ::= & M \mid P|P' \mid \nu z\ P \mid !P \\ M & ::= & 0 \mid \pi.P \mid M + M' \end{array}$$

The structural congruence $\equiv$ is defined as the one for $\pi$ calculus[SW01] with the following three new equivalence for priority guards.

$$\begin{array}{ll} [G : x = G' : x]\pi.P \equiv \pi.P & \text{if } [\![G]\!] \Leftrightarrow [\![G']\!] \\ G : x(y).P \equiv G' : x(y).Q & \text{if } [\![G]\!] \Leftrightarrow [\![G']\!] \text{ and } P \equiv Q \\ G : \overline{x}(H : y).P \equiv G' : \overline{x}(H' : y).Q & \text{if } [\![G]\!] \Leftrightarrow [\![G']\!],\ [\![H]\!] \Leftrightarrow [\![H']\!] \text{ and } P \equiv Q \end{array}$$

Given a process $P$ and a name logic formula $\phi$, $P$ $\mathsf{off}$ $\phi$ is defined as the smallest relation satisfying the following rules. If $P$ $\mathsf{off}$ $\phi$ is the case, $P$ *offers* the set of names. This means the set of names that satisfy $\overline{\phi}$ has the capability to communicate with $P$.

$$G{:}\overline{x}\langle H{:}y\rangle.P \ \mathsf{off}\ \neg[\![G]\!] \wedge \overline{x} \qquad G{:}x(y).P \ \mathsf{off}\ \neg[\![G]\!] \wedge x \qquad \frac{P \ \mathsf{off}\ \phi, Q \ \mathsf{off}\ \psi}{P + Q \ \mathsf{off}\ \phi \vee \psi} \qquad \frac{P \ \mathsf{off}\ \phi, Q \ \mathsf{off}\ \psi}{P|Q \ \mathsf{off}\ \phi \vee \psi}$$

$$\frac{P \ \mathsf{off}\ \phi}{\nu z\ P \ \mathsf{off}\ \phi_{\{z=\mathsf{false}, \overline{z}=\mathsf{false}\}}} \qquad \frac{P \ \mathsf{off}\ \phi}{!P \ \mathsf{off}\ \phi} \qquad \frac{\pi.P \ \mathsf{off}\ \phi}{[g = g']\pi.P \ \mathsf{off}\ \phi}$$

## Operational semantics

We give a basic operational semantics. $P \xrightarrow{\alpha}_{\psi} P'$ means that $P$ evolves to $P'$ after communication of $\alpha$ unless the available communications satisfies $\psi$. Similar to the other calculus with priority, each transition relation $P \xrightarrow{\alpha} P'$ is accompanied with the unavailability condition of names in the environment.

The operational semantics is given by the structural operational semantics shown in figure 1. In the SOS semantics, the compatibility between name logic formulas $\phi$ and $\psi$ is written as $\phi \bowtie \psi$.

For a finite $N$ and $\phi$,$\psi$, when $(N \cup \overline{N}) - \kappa_N^{\mathsf{max}}(\neg\psi) \not\models \phi$ and $(N \cup \overline{N}) - \kappa_N^{\mathsf{max}}(\neg\phi) \not\models \psi$, $\phi \bowtie_N \psi$. If for all finite $N$ $\phi \bowtie_N \psi$, we write $\phi \bowtie \psi$.

In the rules such as $COMM$. $\overline{\phi} \bowtie \psi$ means the environment does not prevent the other party to evolve to the next state, where $\overline{\phi}$ complements all names appearing in $\phi$.

## Further directions

We discuss more concrete examples and the algebraic treatment based on some bisimulation equivalences.

# References


[Phi08] I.C.C. Phillips. CCS with priority guards. *Journal of Logic and Algebraic Programming*, 75(1):139–165, 2008.

[SW01] Davide Sangiorgi and David Walker. *The $\pi$-calculus:A Theory of Mobile Processes*. Cambridge University Press, 2001.

$$\textsc{Tau} \quad \frac{}{S : \tau.P \xrightarrow{\tau}_{[\![S]\!]} P}$$

$$\textsc{Out} \quad \frac{}{S : \overline{x}\langle T : y\rangle.P \xrightarrow{\overline{x}\langle T:y\rangle}_{[\![S]\!]} P} \; \overline{x} \not\models [\![S]\!]$$

$$\textsc{In} \quad \frac{}{S : x(z).P \xrightarrow{x(U:y)}_{[\![S]\!]} P[(U : y)/z]} \; x \not\models [\![S]\!]$$

$$\textsc{Comm} \quad \frac{P \xrightarrow{\overline{x}\langle U:y\rangle}_{\phi} P',\; Q \xrightarrow{x(U:y)}_{\psi} Q',\; P \text{ off } \psi',\; Q \text{ off } \phi'}{P \mid Q \xrightarrow{\tau}_{\phi\vee\psi} P' \mid Q'} \; \phi \bowtie \overline{\phi'}, \psi \bowtie \overline{\psi'}$$

$$\textsc{Par-L} \quad \frac{P \xrightarrow{\alpha}_{\phi} P',\; Q \text{ off } \phi'}{P \mid Q \xrightarrow{\alpha}_{\phi} P' \mid Q} \; \phi \bowtie \overline{\phi'}, \mathsf{bn}(\alpha) \cap \mathsf{fn}(Q) = \emptyset$$

$$\textsc{Sum-L} \quad \frac{P \xrightarrow{\alpha}_{\phi} P'}{P + Q \xrightarrow{\alpha}_{\phi} P'}$$

$$\textsc{Close-L} \quad \frac{P \xrightarrow{\overline{x}\langle U:z\rangle}_{\phi} P',\; Q \xrightarrow{x(U:(z))}_{\psi} Q',\; P \text{ off } \psi',\; Q \text{ off } \phi'}{P|Q \xrightarrow{\tau}_{\phi\vee\psi} \nu z\ (P'|Q')} \; z \notin \mathsf{fn}(Q), \phi \bowtie \overline{\phi'}, \psi \bowtie \overline{\psi'}$$

$$\textsc{Res} \quad \frac{P \xrightarrow{\alpha}_{\phi} P'}{\nu z\ P \xrightarrow{\alpha}_{\phi_{\{z=\mathsf{false},\overline{z}=\mathsf{false}\}}} \nu z\ P'} \; z \notin \mathsf{n}(\alpha)$$

$$\textsc{Open} \quad \frac{P \xrightarrow{\overline{x}\langle U:z\rangle}_{\phi} P'}{\nu z\ P \xrightarrow{\overline{x}\langle U:(z)\rangle}_{\phi} P'} \; z \notin \{x\} \cup \mathsf{n}(U)$$

$$\textsc{Rep-act} \quad \frac{P \xrightarrow{\alpha}_{\phi} P'}{!P \xrightarrow{\alpha}_{\phi} P'|!P}$$

$$\textsc{Rep-comm} \quad \frac{P \xrightarrow{\overline{x}\langle U:z\rangle}_{\phi} P',\; P \xrightarrow{x(U:z)}_{\phi'} P'',\; P \text{ off } \psi}{!P \xrightarrow{\tau}_{\phi\vee\phi'} (P'|P'')|!P} \; \phi \bowtie \overline{\phi'}, (\phi \vee \phi') \bowtie \overline{\psi}$$

$$\textsc{Rep-close} \quad \frac{P \xrightarrow{\overline{x}\langle U:(z)\rangle}_{\phi} P',\; P \xrightarrow{x(U:z)}_{\phi'} P'', P \text{ off } \psi}{!P \xrightarrow{\tau}_{\phi\vee\phi'} (\nu z\ (P'|P''))|!P} \; z \notin \mathsf{fn}(P) \cup \mathsf{n}(U), \phi \bowtie \overline{\phi'}$$

Figure 1: Structural Operational Semantics