Model Independent Order Relations for Processes ⋆

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Abstract. Semantic preorders between processes are usually applied in practice to model approximation or implementation relationships. For interactive models these preorders depend crucially on the observational behaviours of processes as well as on the observing power of environments. The paper aims at a model independent observational theory of the semantic preorders for interactive models. Depending on whether environments change dynamically or not, two classes of model independent preorders are formalized. These formalizations are intensively studied in the framework of CCS. Operational characterizations of these preorders are investigated, and the relationships between them are revealed. Several new preorders for CCS are proposed along the way. Behavioural properties are discussed in a model independent manner as far as possible.

1 Introduction

Observational theory is an old and fruitful area of fundamental research in process calculi, which studies behavioural equivalences and preorders between processes. The starting point is to give clear criteria when one process is a correct implementation or an approximation of the other. It is realized from the very beginning [11, 23] that equivalences or preorders for processes ought to be 'observational' since it is the effect that processes place on environments that really matter. However, one of the insights that has been gained in the past three decades is that there does not really exist the canonical notion of 'observable behaviour'. Depending on different formalizations of observability, many different notions of behavioural equivalences or preorders come out. The readers may consult Van Glabbeek [25, 26] for an overview.

A preorder and its inverse can be combined into an equivalence. Apart from this, preorders on processes have significance of there own. In practice, to build a

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system, we have a specification, say P, which is described as a process in a certain language. Then, this specification is usually implemented as a process, say Q, in the same language. A question one may raise is whether the implementation Q is favourable for the specification P. On such occasions, symmetry is less important. The answer is finding a proper preorder and checking whether P is less than Q.

Historically, there are two families of preorders which turn out to be very successful under two different presumptions on the observation power of environments. They are the testing preorders and the bisimulation equivalences.

In the philosophy of testing theory developed by De Nicola and Hennessy [17], the behaviours of processes are investigated by a series of tests. The preorder between two processes is formulated in terms of their capability to respond to a test. The proposed implementation Q will be considered less than the given specification P whenever Q has all the concerned capabilities that P has. According to two different meanings of 'capability', *may-testing* and *must-testing* preorders are defined respectively. The presumption in the testing approach is the 'static' environments, which means that a test is performed by a single tester in an exclusive manner. Under this presumption, the only thing to concern is the results of tests — success or failure. It does not matter what a process will turn into after a test. Consequently, testing approach will not cope with dynamically changed environments.

In 'dynamic' environments, a process could be subject to interference a potentially unbounded number of observers in an interleaving manner. 'Dynamic' here means that the testers can be dynamically changed during a single test. This scenario usually happens in distributed systems. In this situation, the appropriate process preorders are bisimilarities [20, 11, 10, 12, 27], for the 'bisimulation property' not only cares about testing results, but imposes additional restraints to intermediate states as well. These additional restraints can be conceived as the additional requirements whenever testers are changed. According to the philosophy of bisimilarity, environments have the power to exchange the roles of processes for comparison. Such a strong assumption on the power of environments is considered a shortcoming of the bisimulation approach by some researchers.

This paper is devoted to create a unified observational theory for interactive models, highlighting approximation or implementation relationships, concentrated on two preassumptions about observing power of environments mentioned above. Before further exploration, two beneficial questions ought to be answered at first, from which some important notions are introduced which will be crucial to the characterizations of preorders throughout this article.

The first question is the reasons behind the great success of the testing and the bisimulation approaches. In the opinion of the author, there are two important reasons. The first reason is that the preorders defined in these two approaches support the way of comparing processes in a modular fashion. Intuitively, whenever Q is an approximation of P and they are treated as certain component of an environment $\mathbf{C}[\_]$, a desired property is $\mathbf{C}[Q]$ being an approximation of $\mathbf{C}[P]$. Borrowed from [6], this property is called *extensionality.* It turns out that testing preorders and bisimilarities are all extensional preorders. The second reason, which is more important, is that these preorders can be characterized without depending on the features of special models such as the existence of *labeled transition semantics.* Preorders with this property are called having *model-independent characterizations.* To clarify the precise meaning of 'model-independent' here, we have the following basic requirements on every interaction models. Firstly, every models are supposed to support computations and observations. The former are formalized via reductions or τ-transitions, while the latter are made only by interactions, which are conducted via *interfaces* formalized as *names.* Secondly, in order to support observation, the *composition* $\_\,|\,\_$ and *localization* $(a)(\_)$ operations are indispensable. Composition operation enables observations while localization operation disables observations. Model-independent characterization is motivated to give a profound understanding of observational preorders. With this crucial property, some preorders can be immediately generalized to other interactive models such as name-passing calculi [9] (like π-calculus [13, 14]), value-passing calculi [7] (like value-passing CCS [12]), and process-passing (higher order) calculi (like CHOCS [24], HOπ-calculus [22]).

Behavioural preorders reflect approximating or implementing relations, therefore some 'capabilities' need to be preserved by preorders. Depending on different meanings of approximation or implementation, different notions of 'capabilities' should be concerned, which ultimately lead to different preorders. In this paper, the definitions of preorders are based on preservations of 'capabilities', which leads to the second question: how will 'capabilities' be chosen? We will take the very simple answer: all involved capabilities must be composed from the following four basic ones.

1. $\Diamond$-*capability*: possibly interacting with environment;
2. $\blacksquare$-*capability*: possibly not interacting with environment;
3. $\blacklozenge$-*capability*: impossibly interacting with environment;
4. $\Box$-*capability*: inevitably interacting with environment.

These four basic capabilities will prove powerful enough to produce the most significant preorders. In fact, $\Diamond$-capability and $\Box$-capability are the simplified and model-independent versions for may-testing and must-testing, while $\blacklozenge$-capability and $\blacksquare$-capability are the negations. These four capabilities can be composed arbitrarily to obtain totally 2^4=16 combinations. Following the idea of testing approach, every combination of capabilities will lead to a preorder (not necessarily different). Following the idea of bisimulation approach, however, all these combinations appear redundant, since the bisimulation property covers all four basic capabilities. In literatures, there is another well-known preorder: *similarity*, which lies between bisimilarity and may-testing preorder. Unlike bisimilarity, similarity merely preserves $\Diamond$-capability. One can expect the existence of some other simulation-like preorders which preserve more capabilities above. The 'simulation' approach seems not well-developed before. Some supplements need to be given.

In the present paper, a number of preorders will be formalized. These preorders are defined and studied in a model-independent manner at first. Then the corresponding operational counterparts of all these preorders are investigated in the framework of CCS [11, 12]. The preorders for static environments turn out to be certain variants of testing preorders. Amongst them, failure preorder is redefined so as to cater for the model-independent counterpart, which have more favourable properties. For dynamic environments, it becomes more interesting that the traditional similarity can not be obtained, instead several new simulation-like preorders are discovered during the exploration of operational counterparts, especially t-conserving similarity and f-conserving similarity. The separation results for all these preorders are established for CCS. Moreover, behavioural properties such as stuttering-property, X-property, and computation property are redefined and studied for preorders. Unlike the case for equivalences, stuttering-property and X-property for preorders stress different aspects of process behaviours and they are not able to derive each other.

The rest of the paper is organized as follows. Section 2 lays down the prerequisites of CCS and basic notions for model-independent characterization. Section 3 expounds model-independent characterization of preorders with their behavioural properties and the operational definitions for CCS under the assumption of static environments. Section 4 works in the same framework under the assumption of dynamic environments. Section 5 is the conclusion.

2 Basic Definitions and Notations

2.1 CCS

We begin with the syntax and semantics of CCS. To describe the interactions between systems, we need *names*. The set of the names $\mathcal{N}$ is ranged over by a, b, c, d, e. The set of the names and the conames $\mathcal{L} = \mathcal{N} \cup \overline{\mathcal{N}}$ is ranged over by l and satisties the identity $\overline{\overline{a}} = a$. The set of finite string of names and conames, $\mathcal{L}^*$, is ranged over by u, v, w, s, t, r, and satisfies the identity $\overline{u \cdot v} = \overline{u} \cdot \overline{v}$. To define the operational semantics, we need *action labels*. The set of the action labels $\mathcal{A} = \mathcal{L} \cup \{\tau\}$ is ranged by λ. To introduce infinite behaviours of systems, we introduce the set $\mathcal{C}$ of *constant processes* which is ranged over by A, B, C.

The set $\mathcal{P}$ of CCS processes, ranged over by P, Q, R, M, N, is generated inductively by the following grammar.

$$P \ ::= \ \mathbf{0} \mid \lambda.P \mid P \,|\, P' \mid (a)P \mid P + P' \mid \mathbf{rec}_{C_n}\{C_i \stackrel{\text{def}}{=} P_i\}_{i \in I}$$

We have left out the relabeling operation for two reasons. One interest in CCS is that it is the core language such that the results obtained in CCS can be easily transferred to other interactive models, such as π-calculus. For that purpose the relabeling operation is not necessary. In additional, adding relabeling operator would not make CCS more expressive if infinite behaviours of processes are specified by *constant definition*, see [8] for more on expressiveness of CCS. The binary choice $P + P'$ will be used in its guarded form, meaning that both P and

$$\text{Prefix}\ \frac{}{\lambda.E \stackrel{\lambda}{\longrightarrow} E} \qquad \text{Localization}\ \frac{E \stackrel{\lambda}{\longrightarrow} E' \quad a \text{ does not appear in } \lambda}{(a)E \stackrel{\lambda}{\longrightarrow} (a)E'}$$

$$\text{Choice}\ \frac{E \stackrel{\lambda}{\longrightarrow} E'}{E + F \stackrel{\lambda}{\longrightarrow} E'} \qquad \text{Composition}\ \frac{E \stackrel{\lambda}{\longrightarrow} E'}{E \,|\, F \stackrel{\lambda}{\longrightarrow} E' \,|\, F} \quad \frac{E \stackrel{\alpha}{\longrightarrow} E' \quad F \stackrel{\overline{\alpha}}{\longrightarrow} F'}{E \,|\, F \stackrel{\tau}{\longrightarrow} E' \,|\, F'}$$

$$\text{Constant}\ \frac{P_n\{(\mathbf{rec}_{C_j}\{C_i \stackrel{\text{def}}{=} P_i\}_{i\in I})/C_j\}_{j\in I} \stackrel{\lambda}{\longrightarrow} P'}{\mathbf{rec}_{C_n}\{C_i \stackrel{\text{def}}{=} P_i\}_{i\in I} \stackrel{\lambda}{\longrightarrow} P'}$$

Fig. 1. The Semantics for CCS

P' are in prefix form. The guardedness guarantees the finite branching property. A name a in localization form $(a)P$ is *local*. A name is *global* if it is not local. The notation $\mathsf{gn}(\_)$ $(\mathsf{ln}(\_))$ stands for a function that returns the set of global names (local names). In the form of constant definition $\mathbf{rec}_{C_n}\{C_i \stackrel{\text{def}}{=} P_i\}_{i\in I}$, every constant process in P_i is required to be C_j for some $j \in I$.

The standard semantics of CCS is given by the *labeled transition system* $(\mathcal{P}, \mathcal{A}, \longrightarrow)$. The relation $\longrightarrow \subseteq \mathcal{P} \times \mathcal{A} \times \mathcal{P}$ is the *transition* relation. The membership $(P, \lambda, P') \in \longrightarrow$ is always indicated by $P \stackrel{\lambda}{\longrightarrow} P'$. The relation $\longrightarrow$ is generated inductively by the rules defined in Fig. 1.

The *weak transition* $\Longrightarrow \subseteq \mathcal{P} \times \mathcal{A} \times \mathcal{P}$ is defined as usual: $P \stackrel{\lambda}{\Longrightarrow} P'$ if $P \stackrel{\tau}{\longrightarrow}^{*} \stackrel{\lambda}{\longrightarrow} \stackrel{\tau}{\longrightarrow}^{*} P'$. In the following, both $\longrightarrow$ and $\Longrightarrow$ are lifted as a subset of $\mathcal{P} \times \mathcal{A}^* \times \mathcal{P}$. $P \stackrel{\epsilon}{\Longrightarrow} P'$ is usually abbreviated as $P \Longrightarrow P'$. The mapping $\widehat{\cdot} : \mathcal{A}^* \to \mathcal{L}^*$ is defined by $\widehat{l} = l$, $\widehat{\tau} = \epsilon$, and $\widehat{u \cdot v} = \widehat{u} \cdot \widehat{v}$. We write $P \stackrel{u}{\Longrightarrow}$ if $P \stackrel{u}{\Longrightarrow} P'$ for some P', and write $P \Downarrow$ if $P \stackrel{l}{\Longrightarrow}$ for some l.

It is convenient to define the composed label string. Let $u, u' \in \mathcal{L}^*$. The *composed label string* caused by u and u', denoted by $u_1 \bowtie u_2$ is the set inductively defined as follows.

$$\epsilon \bowtie \epsilon \stackrel{\text{def}}{=} \{\epsilon\}$$

$$\left.\begin{array}{r}(l \cdot u) \bowtie \epsilon \\ \epsilon \bowtie (l \cdot u)\end{array}\right\} \stackrel{\text{def}}{=} \{l\} \cdot (u \bowtie \epsilon)$$

$$(l \cdot u) \bowtie (l' \cdot u') \stackrel{\text{def}}{=} \begin{cases} \{l\} \cdot (u \bowtie (l' \cdot u')) \cup \{l'\} \cdot ((l \cdot u) \bowtie u'), & l \neq \overline{l'} \\ \{l\} \cdot (u \bowtie (l' \cdot u')) \cup \{l'\} \cdot ((l \cdot u) \bowtie u') \cup u \bowtie u', & l = \overline{l'} \end{cases}$$

Intuitively, If $P \stackrel{u}{\Longrightarrow}$ and $P' \stackrel{u'}{\Longrightarrow}$, then $P \,|\, P' \stackrel{w}{\Longrightarrow}$ for every $w \in u \bowtie u'$. Let $V, V' \subseteq \mathcal{L}^*$, $V \bowtie V'$ is defined as $\bigcup_{u\in V, u'\in V'}\{u \bowtie u'\}$.

Lemma 1. *$\bowtie$ is monotone. That is, $V \bowtie W \subseteq V' \bowtie W$ whenever $V \subseteq V'$.*

2.2 Basic Notions for Model-independent Characterization

In Section 1, it is suggested that a process preorder need to be extensional.

Definition 1 (extensionality). *A binary relation* $\mathcal{R}$ *is* extensional *if both the following two statements are valid:*

- *If* $M\mathcal{R}N$ *and* $P\mathcal{R}Q$ *then* $(M \mid P)\mathcal{R}(N \mid Q)$*;*
- *If* $P\mathcal{R}Q$ *then* $(a)P\mathcal{R}(a)Q$*.*

The first statement tells us that, if M is an approximation of N and P is an approximation of Q, then the result of M observing P should be an approximation of the result of N observing Q. The second property confirms that, if P is an approximation of Q, then the approximation relationship is preserved when observation through some ports are prohibited.

Extensionality defined in Definition 1 is to some extent similar to the notion of *pre-congruence* in literatures. Extensionality emphasizes the aspects of observation and interaction, while pre-congruence emphasizes the algebraic aspects. The former is model-independent while the latter is model-dependent. By the way, bisimilarity is extensional yet not a pre-congruence for CCS.

Extensionality suggests that, approximation relationship will be preserved when the related processes are put into a certain environment.

Definition 2 (environment). *An environment* $\mathbf{C}[\_]$ *is either* $[\_]$*, or* $(c)\mathbf{C}'[\_]$*, or* $P \mid \mathbf{C}'[\_]$*, or* $\mathbf{C}'[\_] \mid P$*, where* $c \in \mathcal{N}$*,* $P \in \mathcal{P}$ *and* $\mathbf{C}'[\_]$ *is an environment.*

Lemma 2. *If* $\mathcal{R}$ *is reflexive and extensional, then* $\mathbf{C}[P]\mathcal{R}\mathbf{C}[Q]$ *for every environment* $\mathbf{C}[\_]$ *whenever* $P\mathcal{R}Q$*.*

As is discussed in Section 1, a process preorder may preserve several capabilities. Four basic capabilities are formalized as follows.

Definition 3 (capabilities). *Let* P *be a process.*

1. P *has* $\lozenge$-capability *if* $P \Downarrow$*.*
2. P *has* $\blacksquare$-capability *if for some* P'*,* $P \Longrightarrow P'$ *and* $P' \not\Downarrow$*.*
3. P *has* $\blacklozenge$-capability *if* P *does not have* $\lozenge$*-capability.*
4. P *has* $\square$-capability *if* P *does not have* $\blacksquare$*-capability.*

The preservation of capabilities is defined automatically.

Definition 4 (capability-preservation). *Let* $M \subseteq \{\lozenge, \blacksquare, \blacklozenge, \square\}$ *be a set of basic capabilities. A binary relation* $\mathcal{R}$ *is* M-preserving *if for every* $\bullet \in M$*,* Q *has* $\bullet$*-capability whenever* $P\mathcal{R}Q$ *and* P *has* $\bullet$*-capability.*

$\mathcal{R}$ *is* M-equipollent *if whenever* $P\mathcal{R}Q$*,* Q *has* $\bullet$*-capability if and only if* P *has* $\bullet$*-capability for every* $\bullet \in M$*.*

For concision, strings of basic capabilities are often used to indicate subsets of $\{\lozenge, \blacksquare, \blacklozenge, \square\}$. For example, $\{\lozenge, \blacklozenge, \square\}$-preservation (or $\{\lozenge, \blacklozenge, \square\}$-equipollence) is abbreviated as $\lozenge\blacklozenge\square$-preservation (or $\lozenge\blacklozenge\square$-equipollence).

In views of Definition 3, $\blacklozenge$ (or $\square$) is called *duality* of $\lozenge$ (or $\blacksquare$), and vice versa. Let $M \subseteq \{\lozenge, \blacksquare, \blacklozenge, \square\}$. The duality of M, denoted M^D, is the set of members whose dualities are in M.

Some simple inferences following Definition 3 and Definition 4 are listed as the following lemma.

Lemma 3. *Let R be a binary relation. $M \subseteq \{\Diamond, \blacksquare, \blacklozenge, \Box\}$.*

1. *$\mathcal{R}$ is M-preserving if and only if $\mathcal{R}^{-1}$ is M^D-preserving.*
2. *$\mathcal{R}$ is M-equipollent (or equivalently M^D-equipollent) if and only if R is both M-preserving and M^D-preserving.*

Finally, some technical definitions and lemmas are introduced.

Definition 5. *A binary relation $\mathcal{R}$ is* closed *if the following statements are valid:*

- *For each N, $(N \mid P)\, \mathcal{R}\, (N \mid Q)$ whenever $P\mathcal{R}Q$;*
- *$(a)P\, \mathcal{R}\, (a)Q$ whenever $P\mathcal{R}Q$.*

The relationship between the extensional relations and the closed relations is pointed out in the following lemma.

Lemma 4. *The following statements are valid:*

1. *If $\mathcal{R}$ is reflexive and extensional, then $\mathcal{R}$ is closed.*
2. *If $\mathcal{R}$ is closed, then $\mathcal{R}^*$ is extensional.*

When R is known to be a preorder, Lemma 4 confirms that $\mathcal{R}$ is extensional if and only if $\mathcal{R}$ is closed. Thus Lemma 4 gives us an easier method to show extensionality.

When reasoning about preorders, it is often necessary to construct the *extensional closure* operation on a relation.

Definition 6. *The* extensional closure *$\mathcal{R}^\circ$ of a binary relation $\mathcal{R}$ is inductively defined as follows:*

$$
\begin{aligned}
\mathcal{R}_0 &\stackrel{\text{def}}{=} \mathcal{R} \\
&\vdots \\
\mathcal{R}_{i+1} &\stackrel{\text{def}}{=} \mathcal{R}_i \cup \left\{ \begin{array}{l|l} ((a)P, (a)Q) & P\mathcal{R}_iQ \\ (M \mid P, N \mid Q) & M\mathcal{R}_iN \end{array} \right\} \\
&\vdots \\
\mathcal{R}^\circ &\stackrel{\text{def}}{=} \bigcup_{i \in \omega} \mathcal{R}_i
\end{aligned}
$$

Clearly a binary relation $\mathcal{R}$ is extensional if and only if $\mathcal{R} = \mathcal{R}^\circ$.

3 Orders for Static Environments

3.1 Model-independent Definition of Orders

When environment is static, the model-independent preorders are defined merely via the properties of extensionality and some forms of capability-preservation.

Definition 7 (capability-preserving preorders). *Let* $M \subseteq \{\Diamond, \blacksquare, \blacklozenge, \Box\}$*. The* M-preserving preorder $\leqq_M$ *is the largest relation that is reflexive, extensional and* M*-preserving.*

The M-equipollent preorder *is the largest relation that is reflexive, extensional and* M*-equipollent, which equals to* $\leqq_{M \cup M^D}$*.*

The M-preserving equality, $=_M$*, is defined as* $\leqq_M \cap \leqq_M^{-1}$*.*

Definition 7 may look cumbersome at first glance. Firstly, one can check that 'largest' always makes sense. In additional, from Lemma 5, $\leqq_M$ is reflexive and transitive, which is ensured by the fact that the property of M-preserving is preserved under relational composition. Secondly, under static environments, only the preorders $\leqq_M$ need to focus on. Both M-equipollent preorders and M-preserving equalities are redundant, which is ensured by Lemma 6. These two notions are introduced here for the reason that they are necessary under dynamic environments. Thirdly, a direct inference from Definition 7 is $\leqq_M \subseteq \leqq_{M'}$ whenever $M' \subseteq M$. Lemma 6 even confirms stronger results.

Lemma 5. *Let* $M \subseteq \{\Diamond, \blacksquare, \blacklozenge, \Box\}$*. If* $\{\mathcal{R}_i\}_{i \in I}$ *is a family of reflexive, extensional, and* M*-preserving relations, then* $(\bigcup_{i \in I} \mathcal{R}_i)^*$ *is also a reflexive, extensional, and* M*-preserving relation.*

Lemma 6. *Let* $M, M_1, M_2 \subseteq \{\Diamond, \blacksquare, \blacklozenge, \Box\}$*.*

1. $\leqq_{M^D}$ *coincides with* $\leqq_M^{-1}$*.*
2. $\leqq_{M_1 \cup M_2}$ *coincides with* $\leqq_{M_1} \cap \leqq_{M_2}$*.*
3. $\leqq_{M \cup M^D}$ *coincides with* $=_M$*.*

Proof.
1. Only show that $\leqq_{M^D} \subseteq \leqq_M^{-1}$. The other direction can be proved similarly. We will show that $\leqq_{M^D}^{-1}$ is reflexive, extensional, and M-preserving. The reflexiveness and the extensionality of $\leqq_{M^D}^{-1}$ are guaranteed by their being closed under inverse of relations. M-preservation of $\leqq_{M^D}^{-1}$ is guaranteed by Lemma 3.
2. The inclusion $\leqq_{M_1 \cup M_2} \subseteq \leqq_{M_1}$ is a direct consequence of the fact that $\leqq_M \subseteq \leqq_{M'}$ whenever $M' \subseteq M$. Similarly $\leqq_{M_1 \cup M_2} \subseteq \leqq_{M_2}$, hence $\leqq_{M_1 \cup M_2} \subseteq \leqq_{M_1} \cap \leqq_{M_2}$. The remaining task is to show that $\leqq_{M_1} \cap \leqq_{M_2} \subseteq \leqq_{M_1 \cup M_2}$. To this end, we only notice that $\leqq_{M_1} \cap \leqq_{M_2}$ is $M_1 \cup M_2$-preserving.
3. Notice that $=_M$ is defined as $\leqq_M \cap \leqq_M^{-1}$, this result is a direct inference of 1. and 2.

When all the subsets of $\{\Diamond, \blacksquare, \blacklozenge, \Box\}$ are exhausted, a complete lattice containing at most 16 preorders are produced. Lemma 6 confirms that there is no need to explore each of them one by one. For every nonempty M, $\leqq_M$ can be obtained by taking a few steps of conjunction or inversion from $\leqq_\Diamond$ and $\leqq_\blacksquare$. In view of this, as well as the trivial fact that $\leqq_\emptyset = \mathcal{P}^2$, we shall concentrate on $\leqq_\Diamond$ and $\leqq_\blacksquare$ in the following of this section.

3.2 Behavioural Properties

This part aims to study the behavioural properties of $\leqq_M$. The stuttering property, X-property, and computation property for process equivalences are widely known. The X-property is initially described by De Nicola, Montanari and Vaandrager in [18]. The stuttering property can be found in [27]. These fundamental properties are generalized for process preorders in the following definition.

Definition 8 (Stuttering property, X-property and Computation property). *Let $\leqq$ be a binary relation on $\mathcal{P}$. Let $=$ be $\leqq \cap \leqq^{-1}$.*

1. *$\leqq$ has* stuttering property *if the followings hold: (1) whenever $Q_0 \xrightarrow{\tau} Q_1 \xrightarrow{\tau} \cdots \xrightarrow{\tau} Q_n$, $P \leqq Q_0$ and $P \leqq Q_n$, then $P \leqq Q_i$ for every $0 \leq i \leq n$. (2) whenever $P_0 \xrightarrow{\tau} P_1 \xrightarrow{\tau} \cdots \xrightarrow{\tau} P_m$, $P_0 \leqq Q$, $P_m \leqq Q$, then $P_j \leqq Q$ for every $0 \leq j \leq m$.*
2. *$\leqq$ has* X-property *if $P \Longrightarrow \leqq^{-1} Q$ and $Q \Longrightarrow \leqq^{-1} P$ imply $P = Q$.*
   $\leqq$ has inverted X-property *if $P \Longrightarrow \leqq Q$ and $Q \Longrightarrow \leqq P$ imply $P = Q$.*
3. *$\leqq$ has* computation property *if the following holds: whenever $P_0 \xrightarrow{\tau} P_1 \xrightarrow{\tau} \cdots \xrightarrow{\tau} P_m$ and $P_0 \leqq P_m$, then $P_0 = P_1 = \cdots = P_m$.*
   $\leqq$ has inverted computation property *if the following holds: whenever $P_0 \xrightarrow{\tau} P_1 \xrightarrow{\tau} \cdots \xrightarrow{\tau} P_m$ and $P_m \leqq P_0$, then $P_0 = P_1 = \cdots = P_m$.*

Stuttering property for preorder is motivated from the following intuition. If both Q_0 and Q_n have the capabilities of P, and Q_0 can evolve to Q_n by finite steps of computation, then all the intermediate states between Q_0 and Q_n are deemed to have such capabilities. On the other hand, if Q has the capabilities of both P_0 and P_m, and P_0 can evolve to P_m by a finite number of steps of computation, then Q also has the capabilities of all the intermediate states between P_0 and P_m. Stuttering property is quite natural that every eligible observational preorder should satisfy it.

Computation property for preorder is motivated from the following intuition. During the computation from P_0 to P_m, P_m may lose some capabilities of P_0. If, however, P_m indeed has all the concerned capabilities that P_0 has, then all the intermediate states are deemed to be equal. Computation property has the inverted version because it is possible that, during the computation from P_0 to P_m, P_m may acquire some new capabilities. If this happens, and P_0 has all the concerned capabilities that P_m has, then all the intermediate states are equal. In line with the above, computation property (or its inverted version) does not hold automatically for every preorder. Whether computation property holds or not for a given preorder will depend on the capabilities being concerned. Finally, X-property is a generalized version of computation property.

When considering equivalences only, computation property is a special case of stuttering property. For preorders, however, stuttering property and computation property focus on different aspects. They do not imply each other.

Lemma 7. *Let $M_1, M_2 \subseteq \{\Diamond, \blacksquare, \blacklozenge, \Box\}$.*

1. *If stuttering property holds for $\leqq_{M_1}$ and $\leqq_{M_2}$, then it also holds for $\leqq_{M_1 \cup M_2}$ and $\leqq_{M_1^D}$.*
2. *If X-property (or computation property) holds for $\leqq_{M_1}$ and $\leqq_{M_2}$, then it also holds for $\leqq_{M_1 \cup M_2}$.*
3. *If X-property (or computation property) holds for $\leqq_{M_1}$, then it also holds for $=_{M_1}$.*
4. *X-property (or computation property) holds for $\leqq_{M_1}$ if and only if inverted X-property (or inverted computation property) holds for $\leqq_{M_1^D}$.*

Proof. All results can be proved directly by Definition 8 with the fact stated in Lemma 6 that $\leqq_{M^D} = \leqq_M^{-1}$ and $\leqq_{M_1 \cup M_2} = \leqq_{M_1} \cap \leqq_{M_2}$.

Lemma 7 confirms that behaviour properties of $\leqq_M$ may be derived from those of $\leqq_\lozenge$ and $\leqq_\blacksquare$, which is established in Proposition 1.

Proposition 1. *Stuttering property, X-property, and computation property hold for $\leqq_\lozenge$ and $\leqq_\blacksquare$.*

Proof. To show stuttering property for $\leqq_\lozenge$, suppose $Q_0 \xrightarrow{\tau} Q_1 \xrightarrow{\tau} \cdots \xrightarrow{\tau} Q_n$, $P \leqq_\lozenge Q_0$ and $P \leqq_\lozenge Q_n$. Let $\mathcal{R}$ be the relation $\{(P, Q_i) \mid 0 \leq i \leq n\} \cup \leqq_\lozenge$. Then, it is easy to check that $\mathcal{R}^\circ$ is reflexive, extensional, and $\lozenge$-preserving. The other half, as well as stuttering property for $\leqq_\blacksquare$, can be shown in the same way.

To show X-property for $\leqq_\lozenge$, suppose $P \equiv P_0 \xrightarrow{\tau} P_1 \xrightarrow{\tau} \cdots \xrightarrow{\tau} P_m$ and $Q \leqq_\lozenge P_m$. Let $\mathcal{R}$ be the relation $\{(P_i, Q) \mid 0 \leq i \leq m\} \cup \leqq_\lozenge^{-1}$. Then, it is easy to check that $\mathcal{R}^\circ$ is reflexive, extensional, and $\blacklozenge$-preserving, which confirms $P \leqq_\blacklozenge Q$. The other half which confirms $P \leqq_\lozenge Q$ can be shown in the same way. The proof of X-property for $\leqq_\blacksquare$ is similar.

Corollary 1. *$\leqq_M$ has stuttering property for every $M \subseteq \{\lozenge, \blacksquare, \blacklozenge, \square\}$.*

3.3 Operational Counterparts

This part aims to discover the operational counterparts of $\leqq_\lozenge$ and $\leqq_\blacksquare$ for CCS. As a result, $\leqq_\lozenge$ happens to be trace preorder while $\leqq_\blacksquare$ turns out to be an improved variant of failure preorder which we call potential failure preorder.

Definition 9 (trace preorder). *The* trace set *of P, denoted $\mathbf{TR}(P)$, is the set $\{u \in \mathcal{L}^* \mid P \overset{u}{\Longrightarrow}\}$. We say $P \precsim_{\mathrm{Tr}} Q$, if $Q \overset{u}{\Longrightarrow}$ whenever $P \overset{u}{\Longrightarrow}$. $\precsim_{\mathrm{Tr}}$ is called the* trace preorder*. That is, $P \precsim_{\mathrm{Tr}} Q$ if and only if $\mathbf{TR}(P) \subseteq \mathbf{TR}(Q)$. The* trace equivalence*, $\approx_{\mathrm{Tr}}$, is defined as $\precsim_{\mathrm{Tr}} \cap \precsim_{\mathrm{Tr}}^{-1}$.*

To obtain the first main result of this section that $\precsim_{\mathrm{Tr}}$ coincides with $\leqq_\lozenge$, we need two-side inclusions. $\precsim_{\mathrm{Tr}} \subseteq \leqq_\lozenge$ is ensured by checking $\precsim_{\mathrm{Tr}}$ reflexive, extensional, and $\lozenge$-preserving.

Lemma 8. *$\precsim_{\mathrm{Tr}}$ is reflexive, extensional, and $\lozenge$-preserving.*

Proof. To show $\Diamond$-preservation, suppose that $P \precsim_{\mathrm{Tr}} Q$ and $P \Downarrow$. There must be some $l \in \mathbf{TR}(P)$. Since $\mathbf{TR}(P) \subseteq \mathbf{TR}(Q)$, we have $l \in \mathbf{TR}(Q)$ hence $Q \Downarrow$. To show extensionality, by Lemma 4, it is enough to confirm $\precsim_{\mathrm{Tr}}$ closed. The key is showing $\mathbf{TR}(P \,|\, R) \subseteq \mathbf{TR}(Q \,|\, R)$ whenever $\mathbf{TR}(P) \subseteq \mathbf{TR}(Q)$. Notice that $\mathbf{TR}(P \,|\, R)$ is actually $\mathbf{TR}(P) \bowtie \mathbf{TR}(R)$. By Lemma 1, $\mathbf{TR}(P \,|\, R) = \mathbf{TR}(P) \bowtie \mathbf{TR}(R) \subseteq \mathbf{TR}(Q) \bowtie \mathbf{TR}(R) = \mathbf{TR}(Q \,|\, R)$.

The reversed inclusion is proved by taking advantage of the observing power of CCS.

Theorem 1. *For* CCS, $\leqq_{\Diamond} = \precsim_{\mathrm{Tr}}$*. Therefore,* $\leqq_{\blacklozenge} = \precsim_{\mathrm{Tr}}^{-1}$ *and* $=_{\Diamond} = \approx_{\mathrm{Tr}}$.

Proof. By Lemma 8 and Definition 7, $\precsim_{\mathrm{Tr}} \subseteq \leqq_{\Diamond}$ holds. To prove $\leqq_{\Diamond} \subseteq \precsim_{\mathrm{Tr}}$, suppose that $P \leqq_{\Diamond} Q$, and let $u = l_1 l_2 \ldots l_n \in \mathbf{TR}(P)$. We will show $u \in \mathbf{TR}(Q)$. Construct environment $\mathbf{C}[\_] = (\widetilde{a})(\_ \,|\, \overline{l_1}.\overline{l_2}.\ldots.\overline{l_n}.d)$, in which $\widetilde{a}$ indicates all names in $\mathsf{gn}(P) \cup \mathsf{gn}(Q)$ and $d \notin \mathsf{gn}(P) \cup \mathsf{gn}(Q)$ is a fresh name. By extensionality of $\leqq_{\Diamond}$ and Lemma 2, $\mathbf{C}[P] \leqq_{\Diamond} \mathbf{C}[Q]$. Since $P \stackrel{l_1 l_2 \ldots l_n}{\Longrightarrow}$, $\mathbf{C}[P] \stackrel{d}{\Longrightarrow}$, which means $\mathbf{C}[P] \Downarrow$. By $\Diamond$-preserving, $\mathbf{C}[Q] \Downarrow$, which can only be caused by $Q \stackrel{u}{\Longrightarrow}$. Therefore, $u \in \mathbf{TR}(Q)$.

The second main result is that $\leqq_{\blacksquare}$ coincides with a refined version of failure preorder. To give the operational definition precisely, some additional notations are introduced in advance. Let $V \subseteq \mathcal{L}^*$. The *prefix closure* of V, denoted $\downarrow V$, is the set $\{w \in \mathcal{L}^* \,|\, wv \in V \text{ for some } v\}$. The *first label* of V, denoted $\mathsf{First}(V)$, is $\{l \in \mathcal{L} \,|\, lv \in V \text{ for some } v\}$. Let $v \in \downarrow V$, the *remainder* of V after v, denoted $v^{-1}V$, is the set$\{w \in \mathcal{L}^* \,|\, vw \in V\}$.

Definition 10 (potential failure preorder). *Let* $u \in \mathcal{L}^*$ *and* $V \subseteq \mathcal{L}^*$*.* (u, V) *is called a* potential failure pair *of* P*, if there exists* $w \in \downarrow V$ *and* $P \stackrel{uw}{\Longrightarrow} P'$ *for some* P' *such that* $P' \stackrel{v}{\not\Longrightarrow}$ *for every* $v \in w^{-1}V$*. In potential failure pair* (u, V)*,* u *is called the* trace part*, while* V *is called the* refusal part*.*

The set of all potential failure pairs of P *is denoted by* $\mathbf{FL}(P)$*.* $P \precsim_{\mathrm{Fl}} Q$ *if and only if* $\mathbf{FL}(P) \subseteq \mathbf{FL}(Q)$*.* $P \approx_{\mathrm{Fl}} Q$ *if and only if* $\mathbf{FL}(P) = \mathbf{FL}(Q)$*.* $\precsim_{\mathrm{Fl}}$ *is called* potential failure preorder*.* $\approx_{\mathrm{Fl}}$ *is called* potential failure equivalence*.*

Intuitively, the meaning of potential failure pair (u, V) of P can be understood in the following way. At first P reaches a state P'' by performing u. After that, an execution of P'' is considered successful if P'' performs a trace in V. What the refusal part V affirms is the existence of a trap state P', such that P'' may go into the trap state by performing w, a prefix of some trace in V, and this starting of P'' has no way to be extended to any successful executions.

For more understanding about potential failures, some special cases are studied in the following. In the case $V = \emptyset$, $(u, \emptyset) \in \mathbf{FL}(P)$ if and only if $u \in \mathbf{TR}(P)$. In the case $\epsilon \in V$, $(u, V) \notin \mathbf{FL}(P)$ for every P. In the case $V \subseteq \mathcal{L}$, the traditional failure pairs and failure preorders are obtained.

It needs to be pointed out that the traditional failure preorder is not extensional, and another version of failure equivalence defined in Sect. 9.4 of [12] does not satisfy the property $\mathbf{TR}(P) = \{u \mid (u, \emptyset) \in \mathbf{FL}(P)\}$. These two versions, as well as the one in Definition 10, coincide for finite processes. For infinite processes, however, only potential failure preorder enjoys both the properties. It is worth noting that for CCS, potential failure preorder coincides with *fair testing equivalence* [16] or *should testing equivalence* [5, 21].

An important property of potential failure pairs is stated in the next lemma.

Lemma 9. *If* $(uw, V) \in \mathbf{FL}(P)$*, then* $(u, wV) \in \mathbf{FL}(P)$*.*

We are now in a position to establish the second main result which confirms $\precsim_{\mathrm{Fl}}$ coincident with $\leqq_{\blacksquare}$. To show $\precsim_{\mathrm{Fl}} \subseteq \leqq_{\blacksquare}$, it is enough to check $\precsim_{\mathrm{Fl}}$ being reflexive, extensional, and $\blacksquare$-preserving.

Lemma 10. $\precsim_{\mathrm{Fl}}$ *is reflexive, extensional, and* $\blacksquare$*-preserving.*

Proof. To show $\blacksquare$-preservation, suppose that $P \precsim_{\mathrm{Fl}} Q$ and $P \Longrightarrow\not\Downarrow$. Then, $(\epsilon, \mathcal{L}) \in \mathbf{FL}(P)$. Since $\mathbf{FL}(P) \subseteq \mathbf{FL}(Q)$, we have $(\epsilon, \mathcal{L}) \in \mathbf{FL}(Q)$, hence $Q \Longrightarrow\not\Downarrow$.

Extensionality of $\precsim_{\mathrm{Fl}}$ is more difficult to prove. This result can really be shown indirectly by first proving $\precsim_{\mathrm{Fl}}$ coincident with should-testing preorder, and then proving extensionality for should-testing preorder, which is much easier. For integrity, we give a direct proof here. By Lemma 4, it is enough to confirm $\precsim_{\mathrm{Fl}}$ closed. That is, show $\mathbf{FL}(P \mid R) \subseteq \mathbf{FL}(Q \mid R)$ whenever $\mathbf{FL}(P) \subseteq \mathbf{FL}(Q)$. Suppose that $(u, V) \in \mathbf{FL}(P \mid R)$. By Definition 10, $P \mid R \overset{uu'}{\Longrightarrow} P' \mid R'$ for some $u' \in \downarrow V$, such that $P' \mid R' \overset{u''}{\not\Longrightarrow}$ for every $u'' \in u'^{-1}V$. According to the composition rule in Fig. 1, $P \mid R \overset{uu'}{\Longrightarrow} P' \mid R'$ must be caused by $P \overset{s}{\Longrightarrow} P'$ and $R \overset{r}{\Longrightarrow} R'$. That is, $P \overset{s}{\Longrightarrow} P'$ and $Q \overset{r}{\Longrightarrow} Q'$ for some s, r such that $uu' \in s \bowtie r$. Let V_0 be the set $\{t \in \mathcal{L}^* \mid (t \bowtie \mathbf{TR}(R')) \cap u'^{-1}V \neq \emptyset\}$. The crucial insight is $(s, V_0) \in \mathbf{FL}(P)$. Since $\mathbf{FL}(P) \subseteq \mathbf{FL}(Q)$, we also have $(s, V_0) \in \mathbf{FL}(Q)$, meaning that $Q \overset{s}{\Longrightarrow} Q' \overset{s'}{\Longrightarrow} Q''$ for some Q', Q'', and $s' \in \downarrow V_0$, such that $Q'' \overset{s''}{\not\Longrightarrow}$ for every $s'' \in s'^{-1}V_0$. Now, because $Q \overset{s}{\Longrightarrow} Q'$ and $R \overset{r}{\Longrightarrow} R'$ with $uu' \in s \bowtie r$, we have $Q \mid R \overset{uu'}{\Longrightarrow} Q' \mid R'$, and for every $u'' \in s' \bowtie \mathbf{TR}(R')$ which gives rise to $Q' \mid R' \overset{u''}{\Longrightarrow} Q'' \mid R''$, we claim that $Q'' \mid R'' \overset{u'''}{\not\Longrightarrow}$ for every $u''' \in u''^{-1}u'^{-1}V$, which means $(uu', u'^{-1}V) \in \mathbf{FL}(Q \mid R)$ and will imply $(u, V) \in \mathbf{FL}(Q \mid R)$ by Lemma 9. If not the case, we must have $Q' \mid R' \overset{u''}{\Longrightarrow} Q'' \mid R'' \overset{u'''}{\Longrightarrow}$ for some u''' such that $u''u''' \in u'^{-1}V$, which means that there exists some $s'' \in s'^{-1}V_0$ such that $u''' \in s'' \bowtie \mathbf{TR}(R'')$ and $Q'' \overset{s''}{\Longrightarrow}$. This contradicts with the selection of Q''.

The reversed inclusion is also proved by taking advantage of the observing power of CCS.

Theorem 2. *For* CCS, $\leqq_{\blacksquare} = \precsim_{\mathrm{Fl}}$*. Therefore,* $\leqq_{\square} = \precsim_{\mathrm{Fl}}^{-1}$ *and* $=_{\blacksquare} = \approx_{\mathrm{Fl}}$*.*

Proof. By Lemma 10 and Definition 7, $\precsim_{\mathrm{Fl}} \subseteq \leqq_{\blacksquare}$ holds. To prove $\leqq_{\blacksquare} \subseteq \precsim_{\mathrm{Fl}}$, suppose that $P \leqq_{\blacksquare} Q$, and let $(u, V) \in \mathbf{FL}(P)$. We will show $(u, V) \in \mathbf{FL}(Q)$. Let

d be a name not in $\mathsf{gn}(P) \cup \mathsf{gn}(Q)$. Define processes $R_{u,V,d}$ recursively as follows:

$$R_{u,V,d} = \begin{cases} d + R_{u,V-\{\epsilon\},d} & \text{if } u = \epsilon, \epsilon \in V \\ \sum_{l \in \mathsf{First}(V)} \bar{l}.R_{\epsilon,l^{-1}V,d} & \text{if } u = \epsilon, \epsilon \notin V \\ d + \bar{l}.R_{u',V,d} & \text{if } u = lu' \end{cases}$$

Construct environment $\mathbf{C}[\_] = (\widetilde{a})(\_ \,|\, R_{u,V,d})$, $\widetilde{a}$ indicating all names in $\mathsf{gn}(P) \cup \mathsf{gn}(Q) \cup \mathsf{gn}(R_{u,V,d}) - \{d\}$. By extensionality of $\leqq_{\blacksquare}$ and Lemma 2, $\mathbf{C}[P] \leqq_{\blacksquare} \mathbf{C}[Q]$. Since $(u, V) \in \mathbf{FL}(P)$, we have $P \stackrel{u}{\Longrightarrow} P' \stackrel{w_1}{\Longrightarrow} P''$ for some w_1, P', P'' such that $P'' \stackrel{v}{\not\Longrightarrow}$ for every $v \in w_1^{-1}V$. Now, $\mathbf{C}[P] \equiv (\widetilde{a})(P \,|\, R_{u,V,d}) \Longrightarrow (\widetilde{a})(P' \,|\, R_{\epsilon,V,d}) \Longrightarrow (\widetilde{a})(P'' \,|\, R_{\epsilon,w_1^{-1}V,d})$. According to the definition of $R_{u,V,d}$, $P'' \stackrel{v}{\Longrightarrow}$ if and only if $(\widetilde{a})(P'' \,|\, R_{\epsilon,w_1^{-1}V,d}) \Downarrow$. Since already $P'' \stackrel{v}{\not\Longrightarrow}$, we have $(\widetilde{a})(P'' \,|\, R_{\epsilon,w_1^{-1}V,d}) \not\Downarrow$. In summary, $\mathbf{C}[P] \Longrightarrow \not\Downarrow$. Now, by $\blacksquare$-preserving of $\leqq_{\blacksquare}$, $\mathbf{C}[Q] \Longrightarrow \not\Downarrow$. To make this happen, there must be some Q', Q'' and w_2 such that $\mathbf{C}[Q] \equiv (\widetilde{a})(P \,|\, R_{u,V,d}) \Longrightarrow (\widetilde{a})(Q' \,|\, R_{\epsilon,V,d}) \Longrightarrow (\widetilde{a})(Q'' \,|\, R_{\epsilon,w_2^{-1}V,d}) \not\Downarrow$. This computation can only be caused by $Q \stackrel{u}{\Longrightarrow} Q' \stackrel{w_2}{\Longrightarrow} Q''$ with $Q'' \stackrel{v}{\not\Longrightarrow}$ for every $v \in w_2^{-1}V$, which means $(u, V) \in \mathbf{FL}(Q)$.

The proof of Theorem 2 depends crucially on the guarded choice operator. It is the strong enough observing power provided by the choice operator that makes $\leqq_{\blacksquare}$ as strict as $\precsim_{\mathrm{Fl}}$. For those models without choice operator, we conjecture that $\precsim_{\mathrm{Fl}} \subsetneq \leqq_{\blacksquare}$ and $\leqq_{\lozenge} \not\subseteq \leqq_{\blacksquare}$. Another operator used in the proof of Theorem 2 is the constant definition. If some weaker variants of CCS are considered, Theorem 2 may not hold. At present, what we can make sure is that Theorem 2 does hold for finite processes and finite state processes.

By Theorem 1, Theorem 2, and Lemma 6, every preorder defined model independently in Defintion 7 has its operational counterpart for CCS. By the fact that $\precsim_{\mathrm{Fl}} \subseteq \precsim_{\mathrm{Tr}}$, we have the following.

Lemma 11. *For* CCS, $\leqq_{\blacksquare} \subseteq \leqq_{\lozenge}$.

By Lemma 11 and Lemma 6, all the preorders defined in Definition 7 have been studied in the framework of CCS. They are summarized in the diagram of Fig. 2.

By the way, we point out that for CCS, X-property and computation property do not hold for some preorders in Fig 2.

Proposition 2. *For* CCS, *X-property and computation property hold for all the preorders in Fig. 2 except for* $\leqq_{\blacklozenge}$, $\leqq_{\Box}$, *and* $\leqq_{\Box\lozenge}$.

Proof. All the positive results are inferences of Lemma 7 and Proposition 1. As to the negative results, we select to prove the computation property not holding for $\leqq_{\Box\lozenge}$. Let $P_0 \equiv \tau.a + \tau \stackrel{\tau}{\longrightarrow} a \equiv P_1$. We have $P_0 \leqq_{\Box\lozenge} P_1$ but $P_0 \not\leqq_{\blacksquare\blacklozenge} P_1$.

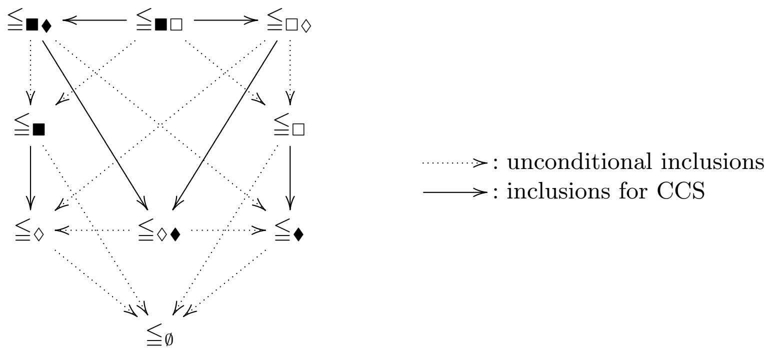

Fig. 2. Model-independent Order Spectrum for CCS: Static Environment

3.4 Separation Results

In this final part, it is confirmed that all the arrows in Fig. 2 indicate strict inclusions.

Proposition 3. *All the inclusions indicated by the arrows in Fig. 2 are strict for* CCS.

Proof. Only consider a few typical inclusions in the left half. When reading the following proof, the readers are supposed to keep in mind the fact that $\leqq_{\lozenge}=\precsim_{\mathrm{Tr}}$ and $\leqq_{\blacksquare}=\precsim_{\mathrm{Fl}}$, etc.

To show $\leqq_{\blacksquare}\subsetneq\leqq_{\lozenge}$ and $\leqq_{\blacksquare\blacklozenge}\subsetneq\leqq_{\lozenge\blacklozenge}$, notice $a+\tau \leqq_{\lozenge\blacklozenge} a$ but $a+\tau \not\leqq_{\blacksquare} a$. To show $\leqq_{\blacksquare\blacklozenge}\subsetneq\leqq_{\blacksquare}$, notice $\tau \leqq_{\blacksquare} \tau+\tau.a$ but $\tau \not\leqq_{\blacksquare\blacklozenge} \tau+\tau.a$. To show $\leqq_{\blacksquare\square}\subsetneq\leqq_{\blacksquare\blacklozenge}$, notice $a \leqq_{\blacksquare\blacklozenge} a+\tau$ but $a \not\leqq_{\blacksquare\square} a+\tau$.

4 Orders for Dynamic Environments

4.1 Model-independent Definition of Orders

In distributed systems, the environment tends to change dynamically. In these situations, additional stronger constraints other than extensionality and some forms of capability-preservation will be imposed on preorders. The constraints concerned in this paper are *simulation property* and *weak simulation property.*

Definition 11 (simulation). *A binary relation $\mathcal{R}$ is a* simulation *if it validates the following* simulation property*: Whenever $P\mathcal{R}Q$ and $P \stackrel{\tau}{\longrightarrow} P'$, then one of the following statements is valid:*

- $Q \Longrightarrow Q'$ *for some Q' such that $P\mathcal{R}Q'$ and $P'\mathcal{R}Q'$.*
- $Q \Longrightarrow Q'' \stackrel{\tau}{\longrightarrow} Q'$ *for some Q'', Q' such that $P\mathcal{R}Q''$ and $P'\mathcal{R}Q'$.*

A binary relation $\mathcal{R}$ is a weak simulation *if it validates the following* weak simulation property*: Whenever $P\mathcal{R}Q$ and $P \stackrel{\tau}{\longrightarrow} P'$, then:*

– $Q \Longrightarrow Q'$ *for some* Q' *such that* $P'\mathcal{R}Q'$.

A binary relation $\mathcal{R}$ *is a* (weak) bisimulation *if both* $\mathcal{R}$ *and* $\mathcal{R}^{-1}$ *are (weak) simulations.* $\mathcal{R}$ *has* (weak) bisimulation property *if both* $\mathcal{R}$ *and* $\mathcal{R}^{-1}$ *has (weak) simulation property.*

By means of extensionality, simulation, and some forms of capability preservation, a set of preorders can be defined model-independently.

Definition 12 (capability preserving simulation preorders). *Let* $M \subseteq \{\Diamond, \blacksquare, \blacklozenge, \Box\}$. *The* M-preserving (weak) simulation preorder $\leqq^{\mathrm{s}}_M$ $(\leqq^{\mathrm{ws}}_M)$ *is the largest (weak) simulation that is reflexive, extensional and* M*-preserving.*

The M-equipollent (weak) simulation preorder *is the largest (weak) simulation that is reflexive, extensional and* M*-equipollent, which equals to* $\leqq^{\mathrm{s}}_{M\cup M^D}$ $(\leqq^{\mathrm{ws}}_{M\cup M^D})$.

The M-preserving (weak) simulation equality, $=^{\mathrm{s}}_M$ $(=^{\mathrm{ws}}_M)$, *is defined as* $\leqq^{\mathrm{s}}_M \cap {\leqq^{\mathrm{s}}_M}^{-1}$ $(\leqq^{\mathrm{ws}}_M \cap {\leqq^{\mathrm{ws}}_M}^{-1})$.

In the first place, all the 'largest' in Defintion 12 must make sense. This is ensured by the following technical lemma.

Lemma 12. *Let* $M \subseteq \{\Diamond, \blacksquare, \blacklozenge, \Box\}$. *If* $\{\mathcal{R}_i\}_{i\in I}$ *is a family of reflexive, extensional, and* M*-preserving (weak) simulations, then* $(\bigcup_{i\in I} \mathcal{R}_i)^*$ *is also a reflexive, extensional, and* M*-preserving (weak) simulation.*

Proof. The crux is that (weak) simulation property is closed under the relational composition. See [2] for the subtlety of this point.

Unlike the situation of static environments, there is no counterpart of Lemma 6 now. What we exactly known is the following lemma.

Lemma 13. *Let* $M, M', M_1, M_2 \subseteq \{\Diamond, \blacksquare, \blacklozenge, \Box\}$.

1. $\leqq^{\mathrm{s}}_M \subseteq \leqq^{\mathrm{ws}}_M \subseteq \leqq_M$.
2. $\leqq^{\mathrm{s}}_M \subseteq \leqq^{\mathrm{s}}_{M'}$ *(or* $\leqq^{\mathrm{ws}}_M \subseteq \leqq^{\mathrm{ws}}_{M'}$*) whenever* $M' \subseteq M$,
3. *If* $\leqq^{\mathrm{s}}_{M_1} = \leqq^{\mathrm{s}}_{M_2}$ *(or* $\leqq^{\mathrm{ws}}_{M_1} = \leqq^{\mathrm{ws}}_{M_2}$*), then* $\leqq^{\mathrm{s}}_{M_1\cup M} = \leqq^{\mathrm{s}}_{M_2\cup M}$ *(or* $\leqq^{\mathrm{ws}}_{M_1\cup M} = \leqq^{\mathrm{ws}}_{M_2\cup M}$*).*
4. *If* $\leqq^{\mathrm{s}}_M$ *(or* $\leqq^{\mathrm{ws}}_M$*) is* M'*-preserving, then* $\leqq^{\mathrm{s}}_{M\cup M'} = \leqq^{\mathrm{s}}_M$ *(or* $\leqq^{\mathrm{ws}}_{M\cup M'} = \leqq^{\mathrm{ws}}_M$*).*

It would be cumbersome to study every $\leqq^{\mathrm{s}}_M$ $(\leqq^{\mathrm{ws}}_M)$ separately. Fortunately, this terrible situation can be improved by Lemma 14 and Lemma 15.

Lemma 14. $\leqq^{\mathrm{s}}_{\Diamond} = \leqq^{\mathrm{ws}}_{\Diamond} = \leqq_{\Diamond}$. $\leqq^{\mathrm{s}}_{\blacksquare} = \leqq^{\mathrm{ws}}_{\blacksquare} = \leqq_{\blacksquare}$.

Proof. Trivially, $\leqq^{\mathrm{s}}_{\Diamond} \subseteq \leqq^{\mathrm{ws}}_{\Diamond} \subseteq \leqq_{\Diamond}$ and $\leqq^{\mathrm{s}}_{\blacksquare} \subseteq \leqq^{\mathrm{ws}}_{\blacksquare} \subseteq \leqq_{\blacksquare}$. Thus, it only need to show $\leqq_{\Diamond} \subseteq \leqq^{\mathrm{s}}_{\Diamond}$ and $\leqq_{\blacksquare} \subseteq \leqq^{\mathrm{s}}_{\blacksquare}$.

To show $\leqq_{\Diamond} \subseteq \leqq^{\mathrm{s}}_{\Diamond}$, it is sufficient to confirm $\leqq_{\Diamond}$ to be an extensional $\Diamond$-preserving simulation. The extensionality and $\Diamond$-preservation hold naturally. The remaining work is to confirm $P' \leqq_{\Diamond} Q$ whenever $P \leqq_{\Diamond} Q$ and $P \Longrightarrow P'$. Namely, Q is able to simulate $P \Longrightarrow P'$ without moving. To this end, let $\mathcal{R}$ be the relation $\{(P', Q) \mid P \leqq_{\Diamond} Q \text{ and } P \Longrightarrow P'\}$. Then, it is not hard to check that $\mathcal{R}^{\circ}$ is reflexive, extensional, and $\Diamond$-preserving. $\leqq_{\blacksquare} \subseteq \leqq^{\mathrm{s}}_{\blacksquare}$ can be shown in the same way.

Lemma 15. *$\leqq^{\mathrm{s}}_{\blacklozenge}$ ($\leqq^{\mathrm{ws}}_{\blacklozenge}$) is $\blacksquare$-preserving. Consequently, $\leqq^{\mathrm{s}}_{\blacklozenge}\subseteq\leqq^{\mathrm{s}}_{\blacksquare}$ ($\leqq^{\mathrm{ws}}_{\blacklozenge}\subseteq\leqq^{\mathrm{ws}}_{\blacksquare}$).*

Proof. Suppose that $P \leqq^{\mathrm{s}}_{\blacklozenge} Q$ (or $P \leqq^{\mathrm{ws}}_{\blacklozenge} Q$) and $P \Longrightarrow P'$ with $P' \not\Downarrow$. By (weak) simulation property, Q' exists such that $Q \Longrightarrow Q'$ and $P' \leqq^{\mathrm{s}}_{\blacklozenge} Q'$ (or $P' \leqq^{\mathrm{ws}}_{\blacklozenge} Q'$). Since $P' \not\Downarrow$, by $\blacklozenge$-preserving, $Q' \not\Downarrow$, which implies $Q \Longrightarrow\not\Downarrow$.

4.2 Behavioural Properties

This part aims to study the behavioural properties of $\leqq^{\mathrm{s}}_{M}$ (or $\leqq^{\mathrm{ws}}_{M}$). In Section 3, stuttering property, X-property, and computation property are studied in a model-independent manner at first. After that, we conclude that it is enough to prove these properties only for $\leqq_{\Diamond}$ and $\leqq_{\blacksquare}$.

This part will take a different way. These properties are only studied in the framework of CCS. Recall that in Section 4.1, the situation is already simplified by Lemma 14 and Lemma 15. Now, the situation is further improved by the following lemma.

Lemma 16. *For* CCS, *$\leqq^{\mathrm{s}}_{\blacksquare}=\leqq^{\mathrm{s}}_{\Diamond\blacksquare}$ ($\leqq^{\mathrm{ws}}_{\blacksquare}=\leqq^{\mathrm{ws}}_{\Diamond\blacksquare}$), $\leqq^{\mathrm{s}}_{\blacklozenge}=\leqq^{\mathrm{s}}_{\Diamond\blacksquare\blacklozenge}$ ($\leqq^{\mathrm{ws}}_{\blacklozenge}=\leqq^{\mathrm{ws}}_{\Diamond\blacksquare\blacklozenge}$), and $\leqq^{\mathrm{s}}_{\Box}=\leqq^{\mathrm{s}}_{\Diamond\blacksquare\blacklozenge\Box}$ ($\leqq^{\mathrm{ws}}_{\Box}=\leqq^{\mathrm{ws}}_{\Diamond\blacksquare\blacklozenge\Box}$). Consequently, for* CCS, *$\leqq^{\mathrm{s}}_{\Diamond}\subseteq\leqq^{\mathrm{s}}_{\blacksquare}\subseteq\leqq^{\mathrm{s}}_{\blacklozenge}\subseteq\leqq^{\mathrm{s}}_{\Box}$ ($\leqq^{\mathrm{ws}}_{\Diamond}\subseteq\leqq^{\mathrm{ws}}_{\blacksquare}\subseteq\leqq^{\mathrm{ws}}_{\blacklozenge}\subseteq\leqq^{\mathrm{ws}}_{\Box}$).*

Proof. $\leqq^{\mathrm{s}}_{\blacksquare}=\leqq^{\mathrm{s}}_{\Diamond\blacksquare}$ is a direct consequence of Lemma 14 and Lemma 11. To show $\leqq^{\mathrm{s}}_{\blacklozenge}=\leqq^{\mathrm{s}}_{\Diamond\blacksquare\blacklozenge}$, we notice that $\leqq^{\mathrm{s}}_{\blacklozenge}=\leqq^{\mathrm{s}}_{\blacksquare\blacklozenge}$ by Lemma 15, together with $\leqq^{\mathrm{s}}_{\blacksquare\blacklozenge}=\leqq^{\mathrm{s}}_{\Diamond\blacksquare\blacklozenge}$ by the fact $\leqq^{\mathrm{s}}_{\blacksquare}=\leqq^{\mathrm{s}}_{\Diamond\blacksquare}$ and Lemma 13. To show $\leqq^{\mathrm{s}}_{\Box}=\leqq^{\mathrm{s}}_{\Diamond\blacksquare\blacklozenge\Box}$, we notice that $\leqq^{\mathrm{s}}_{\Box}=\leqq_{\blacklozenge\Box}$ by the fact $\leqq^{\mathrm{s}}_{\Box}\subseteq\leqq_{\Box}$ being $\blacklozenge$-preserving and Lemma 13, together with $\leqq_{\blacklozenge\Box}=\leqq^{\mathrm{s}}_{\Diamond\blacksquare\blacklozenge\Box}$ by the fact $\leqq^{\mathrm{s}}_{\blacklozenge}=\leqq^{\mathrm{s}}_{\Diamond\blacksquare\blacklozenge}$ and Lemma 13.

Lemma 14 and Lemma 16 tell us that, for CCS, only $\leqq^{\mathrm{s}}_{\blacklozenge}$ ($\leqq^{\mathrm{ws}}_{\blacklozenge}$) and $\leqq^{\mathrm{s}}_{\Box}$ ($\leqq^{\mathrm{ws}}_{\Box}$) require further explorations.

Proposition 4. *For* CCS*, stuttering property, X-property and computation property hold for $\leqq^{\mathrm{s}}_{\blacklozenge}$ ($\leqq^{\mathrm{ws}}_{\blacklozenge}$) and $\leqq^{\mathrm{s}}_{\Box}$ ($\leqq^{\mathrm{ws}}_{\Box}$).*

Proof. The proof of stuttering property is very similar to that of Proposition 1. For example, to show stuttering property holds for $\leqq^{\mathrm{s}}_{\blacklozenge}$, suppose $Q_0 \xrightarrow{\tau} Q_1 \xrightarrow{\tau} \cdots \xrightarrow{\tau} Q_n$, $P \leqq^{\mathrm{s}}_{\blacklozenge} Q_0$ and $P \leqq^{\mathrm{s}}_{\blacklozenge} Q_n$. Let $\mathcal{R}$ be the relation $\{(P, Q_i) \,|\, 0 \le i \le n\} \cup \leqq^{\mathrm{s}}_{\blacklozenge}$. Then, it is easy to check that $\mathcal{R}^{\circ}$ is a reflexive, extensional, and $\blacklozenge$-preserving simulation. The other half of the proof, together with other results, can be shown in the same way.

Proving X-property is more complicated. In the following, the proof is given only for $\leqq^{\mathrm{s}}_{\blacklozenge}$. Suppose $P \equiv P_0 \xrightarrow{\tau} P_1 \xrightarrow{\tau} \cdots \xrightarrow{\tau} P_m$ and $Q \equiv Q_0 \xrightarrow{\tau} Q_1 \xrightarrow{\tau} \cdots \xrightarrow{\tau} Q_n$, with $Q_0 \leqq^{\mathrm{s}}_{\blacklozenge} P_m$ and $P_0 \leqq^{\mathrm{s}}_{\blacklozenge} Q_n$. Let $\mathcal{R}$ be the relation

$$\{(Q_j, P_i) \,|\, 0 \le i \le m \wedge 0 \le j \le n\} \cup \leqq^{\mathrm{s}}_{\blacklozenge}$$

The remaining task is to check that $\mathcal{R}^{\circ}$ is a reflexive, extensional, and $\blacklozenge$-preserving simulation, which means $Q \leqq^{\mathrm{s}}_{\blacklozenge} P$. . The key point here is the $\blacklozenge$-preservation. In other words, we shall confirm $P_i \not\Downarrow$ whenever $Q_j \not\Downarrow$. To this end, assume that $Q_j \Downarrow$. Immediately, $Q_0 \Downarrow$. Due to the fact $Q_0 \leqq^{\mathrm{s}}_{\blacklozenge} P_m$ and $\Diamond$-preservation of $\leqq^{\mathrm{s}}_{\blacklozenge}$ affirmed by Lemma 16, we conclude that $P_m \Downarrow$, which implies $P_i \Downarrow$. The other half which confirms $P \leqq^{\mathrm{s}}_{\blacklozenge} Q$ can be shown in the same way.

The proof of X-property for $\leqq^{\mathrm{s}}_{\blacklozenge}$ ($\leqq^{\mathrm{ws}}_{\blacklozenge}$) and $\leqq^{\mathrm{s}}_{\Box}$ ($\leqq^{\mathrm{ws}}_{\Box}$) depends on $\Diamond$-preservation of $\leqq^{\mathrm{s}}_{\blacklozenge}$ ($\leqq^{\mathrm{ws}}_{\blacklozenge}$) and $\blacksquare$-preservation of $\leqq^{\mathrm{s}}_{\Box}$ ($\leqq^{\mathrm{ws}}_{\Box}$). This is the essential reason why the study of these behaviour properties are limited to CCS.

Combining Proposition 4, Lemma 16, Lemma 14, and Proposition 1, we have the following.

Corollary 2. *For* CCS, *stuttering property, X-property and computation property hold for $\leqq^{\mathrm{s}}_{M}$ ($\leqq^{\mathrm{ws}}_{M}$) whenever $M \subseteq \{\Diamond, \blacksquare, \blacklozenge, \Box\}$.*

4.3 Operational Counterparts

This part aims to discover the operational definitions of $\leqq^{\mathrm{s}}_{M}$ and $\leqq^{\mathrm{ws}}_{M}$ for CCS. Since Lemma 14 and Lemma 16 already inform us that only $\leqq^{\mathrm{s}}_{\blacklozenge}$ ($\leqq^{\mathrm{ws}}_{\blacklozenge}$) and $\leqq^{\mathrm{s}}_{\Box}$ ($\leqq^{\mathrm{ws}}_{\Box}$) are of interest, this part will focus on the operational definitions for $\leqq^{\mathrm{s}}_{\blacklozenge}$ ($\leqq^{\mathrm{ws}}_{\blacklozenge}$) and $\leqq^{\mathrm{s}}_{\Box}$ ($\leqq^{\mathrm{ws}}_{\Box}$). As a result, $\leqq^{\mathrm{s}}_{\blacklozenge}$ ($\leqq^{\mathrm{ws}}_{\blacklozenge}$) coincides with t-conserving (weak) similarity, while $\leqq^{\mathrm{s}}_{\Box}$ ($\leqq^{\mathrm{ws}}_{\Box}$) coincides with f-conserving (weak) similarity.

We begin with the definition of external simulation.

Definition 13 (external simulation). *A binary relation $\mathcal{R}$ over $\mathcal{P}$ is an* external simulation *if whenever $P\mathcal{R}Q$ and $P \xrightarrow{\lambda} P'$, then one of the following statements is valid:*

- *$\lambda = \tau$ and $Q \Longrightarrow Q'$ for some Q' such that $P\mathcal{R}Q'$ and $P'\mathcal{R}Q'$.*
- *$Q \Longrightarrow Q'' \xrightarrow{\lambda} Q'$ for some Q'', Q' such that $P\mathcal{R}Q''$ and $P'\mathcal{R}Q'$.*

A binary relation $\mathcal{R}$ over $\mathcal{P}$ is an external weak simulation *if whenever $P\mathcal{R}Q$ and $P \xrightarrow{\lambda} P'$, then*

- *$Q \stackrel{\widehat{\lambda}}{\Longrightarrow} Q'$ for some Q' such that $P'\mathcal{R}Q'$.*

The operational definition for $\leqq^{\mathrm{s}}_{\blacklozenge}$ ($\leqq^{\mathrm{ws}}_{\blacklozenge}$) is explored now.

Definition 14 (t-conserving similarity). *A binary relation $\mathcal{R}$ over $\mathcal{P}$ is a* t-conserving external (weak) simulation *if $\mathcal{R}$ is an external (weak) simulation, and moreover $\mathbf{TR}(P) = \mathbf{TR}(Q)$ whenever $P\mathcal{R}Q$.*

The t-conserving (weak) similarity, *$\precsim^{\mathrm{s}}_{\mathrm{T=}}$ ($\precsim^{\mathrm{ws}}_{\mathrm{T=}}$), is the largest t-conserving external (weak) simulation. The* t-conserving external (weak) simulation equivalence, *$\approx^{\mathrm{s}}_{\mathrm{T=}}$ ($\approx^{\mathrm{ws}}_{\mathrm{T=}}$), is defined as $\precsim^{\mathrm{s}}_{\mathrm{T=}} \cap {\precsim^{\mathrm{s}}_{\mathrm{T=}}}^{-1}$ ($\precsim^{\mathrm{ws}}_{\mathrm{T=}} \cap {\precsim^{\mathrm{ws}}_{\mathrm{T=}}}^{-1}$).*

The following lemma confirms that t-conserving (weak) similarity is well-defined.

Lemma 17. *If $\{\mathcal{R}_i\}_{i\in I}$ is a family of reflexive t-conserving external (weak) simulations, then $(\bigcup_{i\in I} \mathcal{R}_i)^*$ is also a reflexive t-conserving external (weak) simulation.*

To obtain the first main result of this section that $\precsim^{s}_{T=}$ (or $\precsim^{ws}_{T=}$) coincides with $\leqq^{s}_{\blacklozenge}$ (or $\leqq^{ws}_{\blacklozenge}$), we need two-side inclusions. $\precsim^{s}_{T=} \subseteq \leqq^{s}_{\blacklozenge}$ (or $\precsim^{ws}_{T=} \subseteq \leqq^{ws}_{\blacklozenge}$) is ensured by checking $\precsim^{s}_{T=}$ (or $\precsim^{ws}_{T=}$) a reflexive, extensional, and $\blacklozenge$-preserving (weak) similation.

Lemma 18. $\precsim^{s}_{T=}$ *(* $\precsim^{ws}_{T=}$*) is a reflexive, extensional, and* $\blacklozenge$*-preserving (weak) similation.*

Proof. Only the extensionality deserves brief explanation. By Lemma 4, it is enough to show that $\precsim^{s}_{T=}$ is closed. The key point is to show $(P \mid R) \precsim^{s}_{T=} (Q \mid R)$ whenever $P \precsim^{s}_{T=} Q$. To this end, let $\mathcal{R}$ be the relation $\{(P \mid R, Q \mid R) \mid P \precsim^{s}_{T=} Q\}$. It is obvious that $\mathcal{R}$ is an t-conserving external simulation.

The critical work is to confirm that CCS has strong enough observing power to make $\leqq^{s}_{\blacklozenge}$ ($\leqq^{ws}_{\blacklozenge}$) coincide with $\precsim^{s}_{T=}$ ($\precsim^{ws}_{T=}$).

Theorem 3. *For* CCS, $\leqq^{s}_{\blacklozenge} = \precsim^{s}_{T=}$ *(*$\leqq^{ws}_{\blacklozenge} = \precsim^{ws}_{T=}$*). Therefore,* $=^{s}_{\blacklozenge} = \approx^{s}_{T=}$ *(*$=^{ws}_{\blacklozenge} = \approx^{ws}_{T=}$*).*

Proof. By Lemma 18 and Definition 14, $\precsim^{s}_{T=} \subseteq \leqq^{s}_{\blacklozenge}$ ($\precsim^{ws}_{T=} \subseteq \leqq^{ws}_{\blacklozenge}$) holds. We only need to prove $\leqq^{s}_{\blacklozenge} \subseteq \precsim^{s}_{T=}$. $\leqq^{ws}_{\blacklozenge} \subseteq \precsim^{ws}_{T=}$ can be proved in the same way. For this purpose, we will show that $\leqq^{s}_{\blacklozenge}$ is a t-conserving external simulation.

In order to show that $\leqq^{s}_{\blacklozenge}$ is an external simulation, suppose that $P \leqq^{s}_{\blacklozenge} Q$ and $P \xrightarrow{l} P'$. What we want is two processes Q' and Q'' such that $Q \Longrightarrow Q'' \xrightarrow{l} Q'$ with $P \leqq^{s}_{\blacklozenge} Q''$ and $P' \leqq^{s}_{\blacklozenge} Q'$. Let $\widetilde{a}$ indicates all names in $\mathsf{gn}(P) \cup \mathsf{gn}(Q)$ and $d \notin \mathsf{gn}(P) \cup \mathsf{gn}(Q)$ is a fresh name. Due to extensionality, $P \mid \bar{l}+d \leqq^{s}_{\blacklozenge} Q \mid \bar{l}+d$. Now $P \mid \bar{l}+d \xrightarrow{\tau} P'$, and this computation step should be simulated by $Q \mid \bar{l}+d \Longrightarrow N'$ with $P' \leqq^{s}_{\blacklozenge} N'$. By extensionality, $(\widetilde{a})P' \leqq^{s}_{\blacklozenge} (\widetilde{a})N'$. Now $(\widetilde{a})P' \not\Downarrow$, thus $(\widetilde{a})N' \not\Downarrow$ by $\blacklozenge$-preserving. However $(\widetilde{a})(Q \mid \bar{l}+d) \Downarrow$, meaning that the simulation $Q \mid \bar{l}+d \Longrightarrow N'$ must go like $Q \mid \bar{l}+d \Longrightarrow Q'' \mid \bar{l}+d \xrightarrow{\tau} Q'$ with the property $Q \Longrightarrow Q'' \xrightarrow{l} Q'$, $P' \leqq^{s}_{\blacklozenge} Q'$ and $P \mid (\bar{l}+d) \leqq^{s}_{\blacklozenge} Q'' \mid (\bar{l}+d)$. The remaining work is to confirm $P \leqq^{s}_{\blacklozenge} Q''$. What already known to us is $P \mid (\bar{l}+d) \leqq^{s}_{\blacklozenge} Q'' \mid (\bar{l}+d)$. Owing to extensionality, $P \mid (\bar{l}+d) \mid \overline{d} \leqq^{s}_{\blacklozenge} Q'' \mid (\bar{l}+d) \mid \overline{d}$. This time, using the same technique stated above, $P \mid (\bar{l}+d) \mid \overline{d} \xrightarrow{\tau} P$ can only be simulated by some $Q'' \mid (\bar{l}+d) \mid \overline{d} \Longrightarrow Q''' \mid (\bar{l}+d) \mid \overline{d} \xrightarrow{\tau} Q'''$ with the property $Q \Longrightarrow Q'' \Longrightarrow Q'''$ and $P \leqq^{s}_{\blacklozenge} Q'''$. Finally, by stuttering property, $P \leqq^{s}_{\blacklozenge} Q''$. This is one of the reasons why stuttering property should be established in advance.

In order to show that $\leqq^{s}_{\blacklozenge}$ is t-conserving, it is enough to show $\mathbf{TR}(P) = \mathbf{TR}(Q)$. This is exactly an inference from Theorem 1 and Lemma 13.

The operational definition for $\leqq^{s}_{\Box}$ ($\leqq^{ws}_{\Box}$) is explored now.

Definition 15 (f-conserving similarity). *A binary relation* $\mathcal{R}$ *over* $\mathcal{P}$ *is a* f-conserving external (weak) simulation *if* $\mathcal{R}$ *is an external (weak) simulation, and moreover* $\mathbf{FL}(P) = \mathbf{FL}(Q)$ *whenever* $P\mathcal{R}Q$.

The f-conserving (weak) similarity, $\precsim^{s}_{F=}$ *(*$\precsim^{ws}_{F=}$*), is the largest f-conserving external (weak) simulation. The* f-conserving external (weak) simulation equivalence, $\approx^{s}_{F=}$ *(*$\approx^{ws}_{F=}$*), is defined as* $\precsim^{s}_{F=} \cap {\precsim^{s}_{F=}}^{-1}$ *(*$\precsim^{ws}_{F=} \cap {\precsim^{ws}_{F=}}^{-1}$*).*

The following lemma confirms that f-conserving (weak) similarity is well-defined.

Lemma 19. *If* $\{\mathcal{R}_i\}_{i\in I}$ *is a family of reflexive f-conserving external (weak) simulations, then* $(\bigcup_{i\in I}\mathcal{R}_i)^*$ *is also a reflexive f-conserving external (weak) simulation.*

To obtain the second main result of this section that $\precsim^{\mathrm{s}}_{\mathrm{F=}}$ (or $\precsim^{\mathrm{ws}}_{\mathrm{F=}}$) coincides with $\leqq^{\mathrm{s}}_{\Box}$ (or $\leqq^{\mathrm{ws}}_{\Box}$), we need two-side inclusions. $\precsim^{\mathrm{s}}_{\mathrm{F=}}\subseteq\leqq^{\mathrm{s}}_{\Box}$ (or $\precsim^{\mathrm{ws}}_{\mathrm{F=}}\subseteq\leqq^{\mathrm{ws}}_{\Box}$) is ensured by checking $\precsim^{\mathrm{s}}_{\mathrm{F=}}$ (or $\precsim^{\mathrm{ws}}_{\mathrm{F=}}$) a reflexive, extensional, and $\Box$-preserving (weak) similation.

Lemma 20. $\precsim^{\mathrm{s}}_{\mathrm{F=}}$ *(* $\precsim^{\mathrm{ws}}_{\mathrm{F=}}$*) is a reflexive, extensional, and* $\Box$*-preserving (weak) similation.*

Proof. The extensionality is not easy to prove. By Lemma 4, it is enough to show that $\precsim^{\mathrm{s}}_{\mathrm{F=}}$ is closed. The key point is to show $(P\,|\,R)\precsim^{\mathrm{s}}_{\mathrm{F=}}(Q\,|\,R)$ whenever $P\precsim^{\mathrm{s}}_{\mathrm{F=}}Q$. In order to do this, let $\mathcal{R}$ be the relation $\{(P\,|\,R, Q\,|\,R)\,|\,P\precsim^{\mathrm{s}}_{\mathrm{F=}}Q\}$. The remaining work is to check that $\mathcal{R}$ is an f-conserving external simulation. It is obvious that $\mathcal{R}$ is an external simulation. The rest work is to show $\mathbf{FL}(P\,|\,R)=\mathbf{FL}(Q\,|\,R)$. Because $P\precsim^{\mathrm{s}}_{\mathrm{F=}}Q$, we have $\mathbf{FL}(P)=\mathbf{FL}(Q)$ hence $P\approx_{\mathrm{Fl}}Q$. Now, $\mathbf{FL}(P\,|\,R)=\mathbf{FL}(Q\,|\,R)$ can be obtained by the extensionality of $\approx_{\mathrm{Fl}}$ in Lemma 10.

The critical work is to confirm that CCS has strong enough observing power to make $\leqq^{\mathrm{s}}_{\Box}$ ($\leqq^{\mathrm{ws}}_{\Box}$) coincide with $\precsim^{\mathrm{s}}_{\mathrm{F=}}$ ($\precsim^{\mathrm{ws}}_{\mathrm{F=}}$).

Theorem 4. *For* CCS, $\leqq^{\mathrm{s}}_{\Box}=\precsim^{\mathrm{s}}_{\mathrm{F=}}$ *(*$\leqq^{\mathrm{ws}}_{\Box}=\precsim^{\mathrm{ws}}_{\mathrm{F=}}$*). Therefore,* $=^{\mathrm{s}}_{\Box}=\approx^{\mathrm{s}}_{\mathrm{F=}}$ *(*$=^{\mathrm{ws}}_{\Box}=\approx^{\mathrm{ws}}_{\mathrm{F=}}$*).*

Proof. By lemma 20 and Definition 15, $\precsim^{\mathrm{s}}_{\mathrm{F=}}\subseteq\leqq^{\mathrm{s}}_{\Box}$ ($\precsim^{\mathrm{ws}}_{\mathrm{F=}}\subseteq\leqq^{\mathrm{ws}}_{\Box}$) holds. We only need to prove $\leqq^{\mathrm{s}}_{\Box}\subseteq\precsim^{\mathrm{s}}_{\mathrm{F=}}$. $\leqq^{\mathrm{ws}}_{\Box}\subseteq\precsim^{\mathrm{ws}}_{\mathrm{F=}}$ can be proved in the same way. For this purpose, we will show that $\leqq^{\mathrm{s}}_{\Box}$ is a f-conserving external simulation.

The proof for showing $\leqq^{\mathrm{s}}_{\Box}$ an external simulation goes like that of Theorem 3, therefore omitted. In order to show that $\leqq^{\mathrm{s}}_{\Box}$ is f-conserving, it is enough to show $\mathbf{FL}(P)=\mathbf{FL}(Q)$. This is exactly an inference from Theorem 2 and Lemma 13.

By Theorem 3, Theorem 4, Lemma 14, and Lemma 16, every preorder defined model independently in Defintion 12 has its operational counterpart for CCS.

By the way, when (weak) simulation property is further strengthened to (weak) bisimulation property. We can define *(weak) bisimulation equality* $=^{\mathrm{bis}}$ ($=^{\mathrm{wbis}}$) to be the largest (weak) bisimulation that is reflexive, extensional and M-equipollent. (Weak) Bisimulation equality is an equivalence relation, and it has nothing to do with M provided that $M\neq\emptyset$. For CCS, $=^{\mathrm{bis}}$ ($=^{\mathrm{wbis}}$) coincides with *branching bisimilarity (weak bisimilarity).*

All preorders in this paper for CCS are summarized in the diagram of Fig. 3.

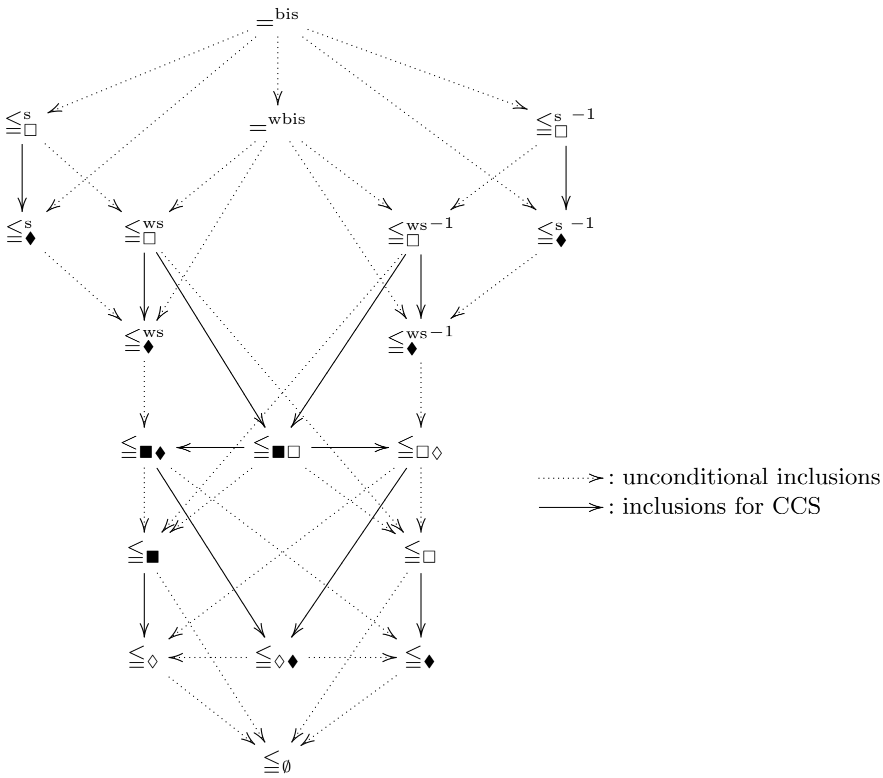

Fig. 3. Model-independent Order Spectrum for CCS

4.4 Separation Results

In this final part, it is confirmed that all the arrows in Fig. 3 indicate strict inclusions.

Proposition 5. *All the inclusions indicated by the arrows in Fig. 3 are strict for* CCS.

Proof. Only consider inclusions in the left half. When reading the following proof, the readers are supposed to keep in mind the fact that $\leqq^{s}_{\blacklozenge} = \precsim^{s}_{T=}$ ($\leqq^{ws}_{\blacklozenge} = \precsim^{ws}_{T=}$) and $\leqq^{s}_{\Box} = \precsim^{s}_{F=}$ ($\leqq^{ws}_{\Box} = \precsim^{ws}_{F=}$).

To show $=^{bis} \subsetneq =^{wbis}$, $\leqq^{s}_{\Box} \subsetneq \leqq^{ws}_{\Box}$, and $\leqq^{s}_{\blacklozenge} \subsetneq \leqq^{ws}_{\blacklozenge}$, let P be $a.(b+\tau)+\tau$ and Q be $a.(b+\tau)$. It is a routine work to check $P =^{wbis} Q$, $P \leqq^{ws}_{\Box} Q$, $P \leqq^{ws}_{\Box} Q$, but $P \neq^{wbis} Q$, $P \nleqq^{ws}_{\Box} Q$, $P \nleqq^{ws}_{\Box} Q$. In fact, $P =^{ws}_{\Box} Q$, $P =^{ws}_{\Box} Q$, thus we have a stronger result that $=^{s}_{\Box} \subsetneq =^{ws}_{\Box}$, and $=^{s}_{\blacklozenge} \subsetneq =^{ws}_{\blacklozenge}$

To show $=^{bis} \subsetneq \leqq^{s}_{\Box}$ or $=^{wbis} \subsetneq \leqq^{ws}_{\Box}$. let $P = \tau.(a+\tau)+\tau.(a+\tau.a+\tau)$ and $Q = \tau.(a+\tau.a+\tau)$. It is a routine work to check $P \leqq^{s}_{\Box} Q$, $P \leqq^{ws}_{\Box} Q$, but $P \neq^{bis} Q$, $P \neq^{wbis} Q$. In fact, $P =^{s}_{\Box} Q$, thus we obtain a stronger result that $=^{bis} \subsetneq \leqq^{s}_{\Box}$, and $=^{wbis} \subsetneq \leqq^{ws}_{\Box}$.

To show $\leqq^{s}_{\Box} \subsetneq \leqq^{s}_{\blacklozenge}$ or $\leqq^{ws}_{\Box} \subsetneq \leqq^{ws}_{\blacklozenge}$. let P be $a + \tau.a + \tau$ and Q be $a + \tau$. It is a routine work to check $P \leqq^{s}_{\blacklozenge} Q$, $P \leqq^{ws}_{\blacklozenge} Q$, but $P \nleqq^{s}_{\Box} Q$, $P \nleqq^{ws}_{\Box} Q$. In fact, $P =^{s}_{\blacklozenge} Q$ and $P =^{ws}_{\blacklozenge} Q$, thus we obtain a stronger result that $=^{s}_{\Box} \subsetneq =^{s}_{\blacklozenge}$ and $=^{ws}_{\Box} \subsetneq =^{ws}_{\blacklozenge}$.

To show $\leqq^{ws}_{\blacklozenge} \subsetneq \leqq_{\blacksquare\blacklozenge}$ or $\leqq^{ws}_{\blacksquare} \subsetneq \leqq_{\blacksquare\Box}$, let P be $a.(\tau.b + \tau.c)$ and Q be $a.b + a.c$. It is a routine work to check $P \leqq_{\blacksquare\Box} Q$ but $P \nleqq^{ws}_{\blacklozenge} Q$.

5 Conclusion

The most significant achievement of this paper is the establishment of a general setting to characterize behavioural preorders between processes model-independently. All those well-known and successful preorders which was once introduced in testing approach or bisimulation approach also have their positions in our setting. Apart from this merit, our approach can serve as a universal platform for defining other valuable preorders and equivalences for interactive models.

Historically, two prototypes of model-independent characterization are the *testing equivalences* [17], presented by R.De Nicola and M.Hennessy, and the *barbed bisimilarity*, presented by R.Milner and D.Sangiorgi [15]. For CCS, may-testing equivalence and must-testing equivalence coincide with trace equivalence and failure equivalence (in Sect. 9.4 of [12]) respectively, while barbed bisimilarity coincide with weak bisimilarity. These two kinds of equivalences are generalized to π-calculus by Boreale and De Nicola [4] and Sangiorgi [22]. For infinite processes, the original must-testing equivalence encounters a well-known problem that it is not contained in trace equivalence. This problem is solved eventually by refining must-equivalence to fair testing equivalence [16] or should-equivalence [5, 21]. For years, these two classes of equivalences only act as two special ways for defining equivalences observationally. Based on the setting developed in this paper, all these equivalences and preorders are defined uniformly. Moreover, some interrelationships between them are revealed, and some other useful preorders are discovered.

Further studies on model-independent characterization may stretch in the following four directions.

The first direction is to present model-independent characterizations for more preorders other than those in this paper. For this purpose, let us have a brief look at linear-time branching-time spectrum [25]. The spectrum were presented at one time as a hierarchy of semantic preorders for LTS. These preorders behave quite well for reactive systems without internal moves, and they can be generalized to processes with internal moves in several different manners [26]. However, it seems that some of these preorders are not adequate for interactive models. The crux is that some robust properties for reactive systems, such as *stability* and *convergence*, become vulnerable for interactive systems, meaning that the composition of two stable or convergent processes could be unstable or divergent. For this reason, imposing additional conditions to those states which have these vulnerable properties would be not very helpful. The equivalences which depend on these properties include *ready equivalence* [19], *ready trace equivalence* [1], *ready simulation equivalence* [3], *failure equivalence* defined in Sect. 9.4 of [12], and *stable bisimilarity*. These kinds Equivalences seem unlikely to have model-independent generalization in the sense of this paper. However, properties such as *instability* and *divergence* are still robust. Accordingly, imposing additional conditions to instable or divergent states are permitted. When restraints such as *codivergence* or *divergence preserving* are added as auxiliary conditions, a set of useful preorders will emerge. The current picture in Fig. 3 will be expanded accordingly. In [9], for instance, four equlities — absolute equality, weak equality, $\Diamond$-equality and $\Box$-equality — are formalized for π-calculus. The operational versions of absolute equality and weak equality restricted in CCS are *codivergent branching bisimilarity* and *codivergent weak bisimilarity* respectively.

The second direction is to re-depict Fig. 3 for some subcalculi of CCS. In [8], the expressiveness of different fragments of CCS are studied. The definition of CCS in this paper is the most general one, which has the strongest observational behaviour as well as the strongest observing power. The observing power is required in showing a preorder defined model-independently contained in the one defined operationally, while observational behaviour is required in showing the other direction. Since the subcalculi may weaken both observational behaviour and observing power simultaneously, the results depicted in Fig. 3 will vary according to the model being concerned. For example, in the proof of Theorem 2, guarded choice are used. If the concerned model did not support guarded choice, Theorem 2 and the inclusion $\leqq_{\Diamond} \subseteq \leqq_{\blacksquare}$ would no longer hold. When this happened, the substitute of Definition 10 should be found. The proof of Theorem 2 also makes use of constant definition. An interesting question is whether Theorem 2 holds if constant definition are replaced with μ-operator, or replication operator.

The third direction is to investigate these preorders in the framework of more general interactive models. Such models include different variations of π-calculus, value passing CCS, and CHOCS, HOπ-calculus. It is controversial which preorders or equivalences are best for these kinds of models. By taking the general setting introduced in this paper, preorders and equivalences can be explored accordingly. Then, operation definitions for them need to study carefully. Works in this direction will shed light on different observational behaviour, different observing power, and different expressiveness for different models.

The fourth direction is to create proof systems for the preorders in the paper. For finite processes, the objective is to establish a uniform setting of proof systems for all reasonable preorders. The author conjectures that both $\leqq^{s}_{\blacklozenge}$ ($\leqq^{ws}_{\blacklozenge}$) and $\leqq^{s}_{\Box}$ ($\leqq^{ws}_{\Box}$) can not be finitely axiomatized. For finite state processes, finding a proof system for $=_{\blacksquare}$ is actually a long standing open problem.

There are lots to do!

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