The Appendix to “On the Canonicity of Higher-order Pi-calculus”

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A Model-independent criteria of encodability

A.0.1 Absolute equality and subbisimilarity

Fu’s model-independent interaction theory [1] is a set of criteria set for encodability of different style compared to Gorla’s [2], which starts from the following basic principles. In a sense, one can read model-independence as general essence of process models. The readers are referred to [1] for more explanation and motivation.

Definition 1. The principles of interaction theory:

- **Object**. There are two kinds of objects, the names and the interactants.
- **Action**. There are two aspects of atomic actions, the internal aspect and the external aspect.
- **Observation**. There are two universal operators, the composition operator and the localization/restriction operator.

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• **Consistency.** A terminating interactant is never equal to a divergent interactant. An observable interactant is never equal to an unobservable interactant.

• **Completeness.** There exists a least expressive model of interaction. This rationale is about interactive completeness. A model is regarded useful only if it is granted such capability.

We then introduce the notion of absolute equality and subbisimilarity [1].

**Absolute equality**

An absolute equality is a relation that is: equipollent, extensional, co-divergence, bisimulation, on the interactants of a model \( M \) of interaction. Each part modifying the absolute equality is defined in the same way as in defining subbisimilarity (Definition 2). The difference between absolute equality and subbisimilarity is that the latter concerns two usually different models, whereas the former is on the same model. It is one of the contribution of Fu to unify the two concepts. Remarkably, the co-divergence property is similar to but stronger than divergence-reflecting, in that it instills the idea of bisimulation into the requirement on the matching of divergent processes.

\( =_{L} \) denotes the absolute equality of process model \( L \). For example, those for \( \Pi \) and \( \pi \) are \( =_{\Pi} \) and \( =_{\pi} \) respectively. As a matter of fact, each model enjoys at least one (external) characterization of \( =_{L} \). Many of the famed bisimilarities in well-known process calculi characterize absolute equalities. For example, the weak bisimilarity in CCS [3], the open bisimilarity in traditional pi-calculus [4] [5]. The context bisimilarity and normal bisimilarity in \( \Pi \) [6] is the characterization of the absolute equality in \( \Pi \) and some of its variants.

It appears one of the central opinions that in each process model, the absolute equality is of canonical significance. In other words, the weak equivalence \( \approx \) in a process language \( (P, \rightarrow, \approx) \) ought to be the corresponding absolute equality of \( P \), with a possible extensional requirement of co-divergence exerted, for example in the definition of weak bisimulation for \( \pi \) in this paper, the co-divergence is absent, thus to comply with absolute equality it should be imposed.

**Subbisimilarity**

The elementary criteria for encodability take as basis the above principles (Definition 1). In this interpretation of encodability named subbisimilarity, a general process model is composed of a set of evolving interactants (processes), the evolving rules (operational semantics) and the absolute equality
(possibly with some characterizations). We sometimes write $\rightarrow^+$ (resp. $\rightarrow^*$) for $\xrightarrow{\tau}$ (resp. $\xRightarrow{\tau}$). The relations in all the discussions here are binary if not stated otherwise.

**Definition 2** (Subbisimilarity). A subbisimilarity is a total relation $\mathcal{T}$ on two models of interaction (say, from $\mathcal{L}$ to $\mathcal{M}$) that is: sound, equipollent, extensional, co-divergent and bisimilar.

- **Soundness.** If $P =_{\mathcal{L}} Q$, then $\mathcal{T}(P) =_{\mathcal{M}} \mathcal{T}(Q)$, where $\mathcal{T}(P)$ is the image of $P$ under $\mathcal{T}$. Soundness is akin to adequacy in Gorla’s criteria [2].

- **Equipollence.** A relation $\mathcal{R}$ is equipollent if whenever $P \mathcal{R} Q$ it holds that
  
  $P \downarrow$ if and only if $Q \downarrow$

  where $P \downarrow$ means $P \downarrow v$ for some $v$.

- **Extensionality.** A relation $\mathcal{R}$ is extensional, if the following property holds:
  
  - If $P \mathcal{R} Q$, then $P | P' \mathcal{R} Q | Q'$ for any $P', Q'$ s.t. $P' \mathcal{R} Q'$;
  
  - If $P \mathcal{R} Q$, then $(c) P \mathcal{R} (c) Q$ for any $c$.

- **Codivergence.** A binary relation $\mathcal{R}$ is co-divergent, if the following property holds whenever $P \mathcal{R} Q$:
  
  - If $P \rightarrow P_1 \rightarrow P_2 \rightarrow \cdots \rightarrow P_i$ where the (internal) (computation) sequence is infinite, then there exist $Q'$ and $j \geq 1$ s.t. $Q \rightarrow^+ Q'$ and $P_j \mathcal{R} Q'$;
  
  - If $Q \rightarrow Q_1 \rightarrow Q_2 \rightarrow \cdots \rightarrow Q_i$ where the (internal) (computation) sequence is infinite, then there exist $P'$ and $j \geq 1$ s.t. $P \rightarrow^+ P'$ and $P' \mathcal{R} Q_j$;

- **Bisimilarity.** A relation $\mathcal{R}$ is a bisimilation, if the following property holds whenever $P \mathcal{R} Q$:
  
  - If $P \rightarrow P'$, then $Q \rightarrow^* Q'$ and $P' \mathcal{R} Q'$;
  
  - If $Q \rightarrow Q'$, then $P \rightarrow^* P'$ and $P' \mathcal{R} Q'$.

Notice name-invariance requires:

$$[P\sigma] = [P]\sigma, \quad (or \ stronger \ [[P\sigma] \equiv [P]\sigma])$$
where $=$ is the absolute equality in the target calculus. It is a primitive in Gorla’s criteria set [2], and actually a derived criterion of subbisimilarity in match/mismatch free calculi. Particularly, combining equipollence, extensionality and bisimulation leads to a similar decree.

There is an assertion on subbisimilarities: *Each subbisimilarity is fully abstract.* The full abstraction is defined in the usual way. Note no requirement of completeness is imposed on the definition of subbisimilarity, since it always holds by the definition. [1] provides more rigid proofs.

References


