Process passing calculus, revisited

Qiang Yin*(尹 强) , Huan Long (龙 环)

(Laboratory of Basic Study In Computing Science,
MOE-MS Key Laboratory for Intelligent Computing and Intelligent Systems,
Department of Computer Science and Engineering, Shanghai Jiaotong University,
Shanghai 200240, China)

Abstract: In the context of process calculi, higher order π calculus (Λ calculus) is prominent and popular due to its ability to transfer processes. Motivated by the attempt to study the process theory in an integrated way, we give a system study of Λ calculus with respect to the model independent framework. We show the coincidence of the context bisimulation to the absolute equality. We also build a subbisimilarity relation from Λ calculus to the π calculus.
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0 Introduction

The process passing calculus, also called higher order π calculus, is proposed by Sangiorgi in Ref. [1] as an extension of the classic mobile calculus : π calculus. Comparing to the other concurrency models, such as Calculus of Communicating System (CCS)[2] and π calculus[3], higher order π calculus is characterized by its ability to transfer processes, which makes it be an efficient mathematical tool for describing and analyzing the mobile systems. Ever since its appearance, there are lots of works about the semantics, algebraic property and expressiveness of higher order π calculus. In Refs. [1, 4, 5] several different semantics have been built, including induction semantics, labeled transition system, etc. Even more bisimulation relations have been proposed, including the famous weak bisimulation, trigger bisimulation, normal bisimulation, etc. At the same time, the

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***E-mail:** yinqiang.sjtu@gmail.com

relationship between the higher order π calculus and other classic calculi, such as λ calculus and π calculus, has also been studied in Ref. [1].

However, a major problem of the previous works, as highlighted in Ref. [6], is that they have all been done in concrete models. As a result it is hard to justify the merits of different semantics and equivalence relations [7]. What is even worse is about the expressiveness study: as the previous studies [8, 9] are proceeded in different frameworks constantly with incomparable criteria, there is hardly any way to judge the reasonableness among different results. In this paper, we amend these problems by revisiting the higher order π calculus in a model independent framework, where the model independent approach is proposed by Fu in Ref. [6] as a consistent framework to study the concurrency theory.

We give a coincidence result of the external bisimulation and the so-called absolute equality in higher order π calculus. The correctness of our characterization is then guaranteed by the model independence of the absolute equality.

We restudy the relative expressiveness of π calculus and higher order π calculus and show that the latter can be encoded into the former in a very strong sense. That is, the encoding satisfies the *subbisimilarity* requirement.

In order to prove the subbisimilarity relation in the above encoding, we have made use of the *up-to expansion* technique. This is the first time that such technique is applied in the study of model independent framework.

1 $\mathit{\Lambda}$ calculus

In this section we give the formal definition of the $\mathit{\Lambda}$ calculus, that is, the process passing calculus we shall work with. The following is the syntax for the processes.

Definition 1 Let X,Y range over process variables, and a,b,...,g,h denote names. The set $\mathcal{T}_{\mathit{\Lambda}}$ of *$\mathit{\Lambda}$-terms* is generated inductively by the following Backus Naur Form (BNF):

$$T \ := \ 0 \ \mid \ X \ \mid \ \sum_{i\in I}\alpha_i.T_i \ \mid \ T\,|\,T' \ \mid \ (c)T \ \mid \ !\alpha.T$$

where I is a finite nonempty indexing set, and the prefix $\alpha_i \ := \ a(X) \ \mid \ \bar{a}(T)$ means input and output actions respectively.

In the above syntax, the inactive process 0 does nothing, $\alpha_i.T_i$ is the sequential operation, and $\sum_{i\in I}\alpha_i.T_i$ is the arbitrary choice within an finite index set

I. The concurrent composition is denoted by the commutative and associative operator "|". The restriction process $(c)T$ creates a new fresh name c within a scope T. The replication $!\alpha.T$ creates as many concurrent replicas of $\alpha.T$ as needed. In $(c)T$ the name c is bound. A name is free if it is not bound. In $a(X).P$ the variable X is bound. A variable is free if it is not bound. We also use $(\widetilde{c})$ to mean $(c_1, c_2 \ldots c_I)$, i.e. a finite set of bound names. The α-conversion applies to both bound names and bound variables. The set of the free names of T is denoted by $fn(T)$, and the set of the free variables in T by $fv(T)$. Similarly we use the notions of $bn(T)$ and $bv(T)$ for bound names and bound variables. The set $\mathcal{P}_\Lambda$ of Λ-*processes* consists of those Λ-terms in which all the term variables are bound. We abbreviate $a(X).T$ as $a.T$ when X in not in $fv(P)$; $\bar{a}(0).T$ as $\bar{a}.T$; $\bar{a}(T).0$ as $\bar{a}(T)$; $a(X).0$ as $a(X)$.

Structure

$$\frac{T \equiv T_1 \quad T \xrightarrow{\lambda} S \quad S \equiv S_1}{T_1 \xrightarrow{\lambda} S_1}$$

Prefix

$$\frac{}{a(X).T \xrightarrow{a(E)} T\{E/X\}} \qquad \frac{}{\bar{a}(T).T' \xrightarrow{\bar{a}(T)} T'}$$

Composition

$$\frac{T \xrightarrow{\lambda} T'}{S|T \xrightarrow{\lambda} S|T'} \qquad \frac{S \xrightarrow{a(E)} S' \quad T \xrightarrow{(\widetilde{c})\bar{a}(E)} T'}{S|T \xrightarrow{\tau} (\widetilde{c})(S'|T')}$$

Localization

$$\frac{T \xrightarrow{(\widetilde{c})\bar{a}(E)} T'}{(d)T \xrightarrow{(\widetilde{c} \bigcup d)\bar{a}(E)} T'} \; d \in (fn(E) - \widetilde{c}) \qquad \frac{T \xrightarrow{\lambda} T'}{(d)T \xrightarrow{\lambda} (d)T'} \; d \notin n(\lambda)$$

Replication $$\frac{\alpha.T \xrightarrow{l} T'}{!\alpha.T \xrightarrow{l} T'|!\alpha.T}$$ Sum $$\frac{\alpha_k.T_k \xrightarrow{l} T'}{\sum_{i \in I} \alpha_i.T_i \xrightarrow{l} T'}$$

Fig.1 The labeled transition system of Λ

In the semantics we require that a term released in an output action must be a process. The operational semantics is given by the labeled transition system in

Fig. 1. Symmetric rules are omitted. The set $\mathcal{L}$ of labels for Λ-terms, ranges over by l, is $\{a(E), \bar{a}(E), (\tilde{c})\bar{a}(E) \mid E \in \mathcal{P}_\Lambda\}$, and λ ranges over the set $\mathcal{L} \cup \{\tau\}$, where τ is the silent action standing for interaction. We write $\Longrightarrow$ for the reflexive, transitive closure of $\xrightarrow{\tau}$, and $\stackrel{\lambda}{\Longrightarrow}$ for $\Longrightarrow\xrightarrow{\lambda}\Longrightarrow$. The notation "$\equiv$" is used to indicate structure congruence.

2 Observational Theory

As we have mentioned in the introduction part, there has been a long history about the arguments over different semantics [7, 10]. In this section we introduce the model independent framework for the study of interactive model, especially the so called absolute equality relation, as a unique criterion for judging semantics. Then we show that the context bisimulation of the Λ calculus coincides with the absolute equality relation.

For an interactive model $\mathbb{M}$, let $\mathcal{P}_\mathbb{M}$ and $\mathcal{N}_\mathbb{M}$ be process set and name set respectively; $\mathcal{P}_\mathbb{M}$ is ranged over by P, Q, M and N; $\mathcal{N}_\mathbb{M}$ is ranged over by a, b, c and d. We say $\mathcal{R}$ is a binary relation on $\mathcal{P}_\mathbb{M}$ if $\mathcal{R} \subseteq \mathcal{P}_\mathbb{M} \times \mathcal{P}_\mathbb{M}$. We use $P\mathcal{R}Q$ to denote that $(P, Q) \in \mathcal{R}$. We say $\mathcal{R}$ is symmetric if $P\mathcal{R}Q$ implies $Q\mathcal{R}P$, and reflexive if $\forall P \in \mathcal{P}_\mathbb{M}$, $(P, P) \in \mathcal{R}$. We use "$\mathcal{R}_1; \mathcal{R}_2$" to denote the composition of two binary relation $\mathcal{R}_1$ and $\mathcal{R}_2$. The next two definitions are from Ref. [6].

Definition 2 A binary symmetric relation $\mathcal{R}$ on $\mathcal{P}_\mathbb{M}$ is a bisimulation if it validates the following bisimulation property:

If $Q\mathcal{R}^{-1}P \xrightarrow{\tau} P'$, then one of the following statements is valid:
(1) $Q \Rightarrow Q'$ for some Q', such that $Q'\mathcal{R}^{-1}P$ and $Q'\mathcal{R}^{-1}P'$;
(2) $Q \Rightarrow Q''\mathcal{R}^{-1}P$ for some Q'', such that $Q'' \xrightarrow{\tau} Q'\mathcal{R}^{-1}P'$.

It is codivergent if the following codivergent property is satisfied:

If $Q\mathcal{R}P\xrightarrow{\tau}P_1\xrightarrow{\tau}\cdots\xrightarrow{\tau}P_i\cdots$ is an infinite τ action sequence, then there exist some Q' and $i \geq 1$ such that $Q\stackrel{\tau}{\Longrightarrow}Q'\mathcal{R}P_i$.

It is *extensional* if the following extensionality property holds:

(1) If $M\mathcal{R}N$ and $P\mathcal{R}Q$, then $(M|P)\mathcal{R}(N|Q)$;
(2) If $P\mathcal{R}Q$, then $(a)P\mathcal{R}(a)Q$ for every $a \in \mathcal{N}_\mathbb{M}$.

Definition 3 The absolute equality "$=_\mathbb{M}$" on $\mathcal{P}_\mathbb{M}$ is the largest relation validating the following statements:

(1) The relation is reflexive;
(2) The relation is equipollent, extensional, codivergent and bisimilar.

The following two lemmas will be used in the sequel.

Lemma 1 (Ref. [6]) If $P \Longrightarrow; =_{\mathbb{M}} Q$ and $Q \Longrightarrow; =_{\mathbb{M}} P$, then $P =_{\mathbb{M}} Q$.

Lemma 2 Suppose $P =_{\mathbb{M}} Q$ and $P \xrightarrow{\tau} P_1 \xrightarrow{\tau} P_2 \xrightarrow{\tau} \cdots \xrightarrow{\tau} P_n \cdots$ is infinite τ action sequence of P, then there is an infinite action sequence of Q

$$Q \xrightarrow{\tau} Q_1 \xrightarrow{\tau} Q_2 \xrightarrow{\tau} \cdots \xrightarrow{\tau} Q_n \cdots ,$$

and an increasing function f of $\mathbb{N}$ such that $P_{f(i)} =_{\mathbb{M}} Q_i \, (i \geq 1)$.

All the properties introduced in Definition 2 are independent of concrete models. For any interaction model $\mathbb{M}$, we can define the absolute equality $=_{\mathbb{M}}$ as above. From another point of view, the absolute equality can be seen as a collection of minimal requirement for a good observational equivalent relation. Let $=_{\Lambda}$ be the largest reflexive equipollent extensional codivergent bisimilar relation on $\mathcal{P}_{\Lambda}$. In the rest of this section we will show that $=_{\Lambda}$ can be established in a more tractable way in terms of a branching version of context bisimulation defined in Ref. [1].

Definition 4 A symmetric relation $\mathcal{R}$ on $\mathcal{P}_{\Lambda}$ is a context bisimulation if it validates the following property:

(1) $\mathcal{R}$ is codivergent and bisimilar;
(2) If $Q\mathcal{R}^{-1}P \xrightarrow{a(E)} P'$, then $Q \Rightarrow Q'' \xrightarrow{a(E)} Q'\mathcal{R}^{-1}P'$ and $P\mathcal{R}Q''$ for some Q',Q'';
(3) If $P \xrightarrow{(\tilde{b})\bar{a}(E)} P'$, then $\forall G$ with $fn(G) \cap \{\widetilde{b}\} = \emptyset$, there exists some $\widetilde{c}$,F,Q' and Q'', such that $Q \Longrightarrow Q'' \xrightarrow{(\tilde{c})\bar{a}(F)} Q'$, and it holds that $fn(G) \cap \{\widetilde{c}\} = \emptyset$, $P\mathcal{R}Q''$ and

$$(\tilde{b})(G[E]|P') \, \mathcal{R} \, (\tilde{c})(G[F]|Q').$$

The context bisimilarity, denoted as $\approx_{\Lambda}$, is the largest context bisimulation on $\mathcal{P}_{\Lambda}$. We say that P,Q are context bisimilar, written as $P \approx_{\Lambda} Q$, if $P\mathcal{R}Q$ for some context bisimulation $\mathcal{R}$.

The following proposition shows that $\approx_{\Lambda}$ is a congruence. More specifically, it is closed under the operation of composition and localization.

Proposition 1 $\forall P, Q \in \mathcal{P}_\Lambda$, if $P \approx_\Lambda Q$, then

(1) $\forall M \in \mathcal{P}_\Lambda$, $P \mid M \approx_\Lambda Q \mid M$;
(2) $(a)P \approx_\Lambda (a)Q$.

Proof We only prove the first item. Define a binary relation $\mathcal{R}$ as follows and we can prove that $\mathcal{R}$ is a context bisimulation up-to $\equiv$ and up-to restriction[1].

$$\mathcal{R} \stackrel{\text{def}}{=} \{(P|M, Q|M) \ \mid \ P \approx_\Lambda Q\}$$

This can almost be done by an analogous argument as mentioned in Ref. [1], so we omit the detail. Here we only discuss the codivergent property.

Suppose $P|M{=}P_0 \xrightarrow{\tau} P_1 \xrightarrow{\tau} P_2 \xrightarrow{\tau} \cdots \xrightarrow{\tau} P_i \xrightarrow{\tau} \cdots$ is an infinite τ sequence, we label all these τ actions by τ_P, τ_M and $\tau_{P,M}$ to indicate that these τ actions are caused by only P, only M , and both P, M respectively. If there is no $\tau_{P,M}$ then by codivergent and bisimilar property it is easy to see that $Q|M \stackrel{\tau}{\Longrightarrow} Q'$ and $P_i \mathcal{R} Q'$ for some $i \geq 1$. Otherwise let P_i be the first process that can perform an $\tau_{P,M}$, then $P_i \equiv P'|M' \xrightarrow{\tau_{P,M}} P_{i+1}$ and by bisimilar we have $Q|M \Longrightarrow Q'|M'$ and $P' \approx_\Lambda Q'$. Use the simple diagram-chasing we can get some Q'' such that $Q'|M' \stackrel{\tau}{\Longrightarrow} Q''$ and $P_{i+1} \mathcal{R} Q''$.

By Definition 4, $\approx_\Lambda$ is reflexive, bisimilar and equipollent. By Proposition 1, $\approx_\Lambda$ is extensional. Then by Definition 3, the following lemma holds.

Lemma 3 $\approx_\Lambda \subseteq =_\Lambda$.

Lemma 3 shows the soundness of the context bisimulation and we are able to prove that, the other direction of the inclusion, $=_\Lambda \subseteq \approx_\Lambda$ also holds. As a result we have the following theorem.

Theorem 1 $\approx_\Lambda$ coincides with $=_\Lambda$.

Proof By Lemma 3 it is sufficient to prove $=_\Lambda \subseteq \approx_\Lambda$. Let

$$\mathcal{R} \stackrel{\text{def}}{=} \{(P, Q) \ \mid \ P =_\Lambda Q\}$$

and we now prove that $\mathcal{R}$ is a context bisimulation. By the definition of $=_\Lambda$, we only need consider the following two cases:

(1) If $P \xrightarrow{a(E)} P_1$. Let C be $\bar{a}(E).0 + \bar{f}$ for some fresh name f. By equivalence and extensionality, $P \mid C \xrightarrow{\tau} P_1$ must be matched by $Q|C \Longrightarrow Q_2|C \xrightarrow{\tau} Q_1$

and $Q_2 \xrightarrow{a(E)} Q_1$ for some Q_1,Q_2. Since $Q_2|C \xrightarrow{\tau} Q_1$ must be a change of state, it must be the case that $P|C =_\Lambda Q_2|C$ and $P_1 =_\Lambda Q_1$. Use the same argument, $P|C|f \xrightarrow{\tau} P$ must be matched by $Q_2|C|f \Longrightarrow Q_3|C|f \xrightarrow{\tau} Q_4$ and

$$P =_\Lambda Q \Longrightarrow Q_2 \Longrightarrow Q_4 =_\Lambda P.$$

It follows from Lemma 1 that $P =_\Lambda Q_2$. In conclusion, we have $Q \Longrightarrow Q_2 \xrightarrow{a(E)} Q$ with $P\mathcal{R}Q_2$ and $P_1\mathcal{R}Q_1$.

(2) If $P \xrightarrow{(\widetilde{c})\bar{a}(E)} P_1$. Let C be $a(X).G[X]+\bar{f}$ for an arbitrary context G with $fn(G)\cap\{\widetilde{c}\}=\emptyset$, and f is a fresh names. By equipollence and extensionality, $P|C \xrightarrow{\tau} (\widetilde{c})(P_1|G[E])$ must be matched by $Q|C \Longrightarrow Q_2|C \xrightarrow{\tau} (\widetilde{d})(Q_1|G[F])$ and $Q_2 \xrightarrow{(\widetilde{d})\bar{a}(F)} Q_1$ for some $\widetilde{d}$ and F. Since $Q_2|C \xrightarrow{\tau} (\widetilde{d})(Q_1|G[F])$ must be a change of state, then it must be the case that $P|C =_\Lambda Q_2|C$ and $(\widetilde{c})(P_1|G[E]) =_\Lambda (\widetilde{d})(Q_1|G[F])$. By α-conversion[2] we can always require $fn(G) \cap\{\widetilde{d}\}=\emptyset$ and use the same argument in Case (1) we have $P =_\Lambda Q_2$. In conclusion, we have $\forall G$ with $fn(G)\cap\{\widetilde{c}\}=\emptyset$, there is $\widetilde{c}$,Q_2,Q_1 with $fn(G)\cap\{\widetilde{d}\}=\emptyset$ such that $Q \Longrightarrow Q_2 \xrightarrow{(\widetilde{c})\bar{a}(F)} Q_1$ and $P\mathcal{R}Q_2$ and

$$(\widetilde{c})(P_1|G[E]) \ \mathcal{R} \ (\widetilde{d})(Q_1|G[F]).$$

This finishes the proof.

Theorem 1 is the main theorem about the algebraic property of the Λ calculus under the model independent framework. The significance of it is that it shows an exact correspondence between the observational theory of concrete model, i.e., the Λ calculus, and the model independent theory. This makes the result important in itself, and it is also very helpful to pursue the following research.

3 Relative Expressiveness

In the section 2 we have revisited the semantics and algebraic property of the Λ calculus under the model independent framework. In this section we will focus on the expressiveness issue of Λ calculus, especially the relative expressiveness with respect to the first order π calculus.

There have been several previous works about the expressiveness issue of the process passing calculus [11–13]. It is well-known that Λ calculus is at most as expressiveness as the π calculus. This fact is formally established by Sangiorgi in

Refs. [1, 14]. Rather than taking the Sangiorgi's result for granted, here we adopt the theory of expressiveness developed in Refs. [6, 15] and reformulate this result by showing that there is a subbisimilarity from Λ calculus to the π calculus. The π calculus we used is presented in Ref. [16] and the following Definition is from Ref. [6].

Definition 5 A relation $\mathfrak{R}$ from $\mathbb{M}_0$ to $\mathbb{M}_1$ is a subbisimilarity, notation $\mathfrak{R} : \mathbb{M}_0 \to \mathbb{M}_1$, if it validates the following statements.

(1) $\mathfrak{R}$ is reflexive in the following sense:

  (i) $\mathfrak{R}$ is total, meaning that for all $P \in \mathcal{P}_{\mathbb{M}_0}$ there exists $Q \in \mathcal{P}_{\mathbb{M}_1}$ such that $P\mathfrak{R}Q$.

  (ii) $\mathfrak{R}$ is sound, meaning that $Q_1\mathfrak{R}^{-1}P_1 =_{\mathbb{M}_0} P_2\mathfrak{R}Q_2$ implies $Q_1 =_{\mathbb{M}_1} Q_2$.

(2) $\mathfrak{R}$ is equipollent, extensional, codivergent and bisimilar.

The following proposition is an easy deduction of the definition of subbisimilarity.

Proposition 2 (Ref. [6]) Subbsimilarities are fully abstract.

We say that $\mathbb{M}_0$ is subbisimilar to $\mathbb{M}_1$, notation $\mathbb{M}_0 \sqsubseteq \mathbb{M}_1$, if there is a subbisimilarity from $\mathbb{M}_0$ to $\mathbb{M}_1$. We write $\mathbb{M}_0 \sqsubset \mathbb{M}_1$ if $\mathbb{M}_0 \sqsubseteq \mathbb{M}_1$ but $\mathbb{M}_1 \not\sqsubseteq \mathbb{M}_0$. It is routine to show that both $\sqsubseteq$ and $\sqsubset$ are transitive. To show that a relation is a subbisimilarity, the most tricky part is to establish the soundness property. However, in our case, by utilizing Theorem 1 and the following theorem, the workload of soundness proof can be greatly decreased .

Definition 6 (Ref. [16]) A codivergent bisimulation on $\mathcal{P}_\pi$ is an external bisimulation if the following statements are valid for every $l \in \mathcal{L}$.

(1) If $Q\mathcal{R}^{-1}P\xrightarrow{l}P'$ then $Q\Longrightarrow Q''\xrightarrow{l}Q'\mathcal{R}^{-1}P'$ and $P\mathcal{R}Q''$ for some Q', Q'';

(2) If $P\mathcal{R}Q\xrightarrow{l}Q'$ then $P\Longrightarrow P''\xrightarrow{l}P'\mathcal{R}Q'$ and $P''\mathcal{R}Q$ for some P', P''.

The π-bisimilarity, donated as $\approx_\pi$, is the largest external bisimulation on $\mathcal{P}_\pi$. We say that P,Q are external bisimilar, written as $P \approx_\pi Q$, if $P\mathcal{R}Q$ for some external bisimulation $\mathcal{R}$.

Theorem 2 (Ref. [16]) The π-bisimilarity $\approx_\pi$ coincides with the absolute equality $=_\pi$.

From Definition 6 we can see that $\approx_\pi$ dose not require the extensionality property, while Theorem 2 tells us that it has been implicitly included in $\approx_\pi$. This makes $\approx_\pi$ more tractable than $=_\pi$. As we have just mentioned, Theorem 2 will be a powerful tool for us to get the fully abstract result, i.e. Theorem 3.

Now we are ready to present our main result about the relative expressiveness issue of Λ calculus under the model independent framework.

3.1 Interpret Λ into π

$$
\begin{aligned}
[\![0]\!] &\stackrel{\text{def}}{=} 0\\
[\![X]\!] &\stackrel{\text{def}}{=} (d)(\bar{x}d|!d)\\
[\![a(X).T]\!] &\stackrel{\text{def}}{=} a(x).[\![T]\!]\\
[\![\bar{a}(T_1).T_2]\!] &\stackrel{\text{def}}{=} (c)(\bar{a}c.([\![T_2]\!]\ |!c.[\![T_1]\!]))\\
[\![(c)T]\!] &\stackrel{\text{def}}{=} (c)[\![T]\!]\\
[\![T_1|T_2]\!] &\stackrel{\text{def}}{=} [\![T_1]\!]|[\![T_2]\!]\\
[\![!\alpha.T]\!] &\stackrel{\text{def}}{=} ![\![\alpha.T]\!]
\end{aligned}
$$

Fig. 2 Translation from Λ into π

The interpretation $[\![\cdot]\!]$ from Λ calculus to π calculus in Ref. [1] is given in Fig. 2. Intuitively, the encoding scheme says that transferring a process in Λ calculus can be simulated by transferring a pointer which will be used to call the process in π calculus. From the above interpretation it is clear that the following proposition holds.

Proposition 3 (Syntactical correctness) $\forall P \in \mathcal{P}_\Lambda \quad [\![P]\!] \in \mathcal{P}_\pi$.

We shall use "Tr_n" as an abbreviation for $(d)(\bar{n}d|!d)$. The utility of Tr_n is to activate a copy of $[\![E]\!]$ from a process with a sub-term as $!n.[\![E]\!]$. In the later, n can be replaced by any names or variables.

The following example shows how the interpretation work does.

Example 1 Let $P = a(X).\bar{b}(E).X$, $Q = \bar{a}(F).D$, then

$$
\begin{aligned}
&P|Q = a(X).\bar{b}(E).X|\bar{a}(F).D \xrightarrow{\tau} \bar{b}(E).F|D,\\
&[\![P|Q]\!] = a(x).(c)(\bar{b}c.(Tr_x|!c.[\![E]\!])) \mid (d)(\bar{a}d.([\![D]\!]|!d.[\![F]\!]))\\
&\qquad \xrightarrow{\tau} (d)((c)(\bar{b}c.Tr_d|!c.[\![E]\!])|[\![D]\!]|!d.[\![F]\!]),\\
&[\![\bar{b}(E).F|D]\!] = (c)(\bar{b}c.[\![F]\!]|!c.[\![E]\!])|[\![D]\!]
\end{aligned}
$$

In the π calculus, we will need the following concept.

Definition 7 (Context) A first-order context G is defined inductively as follows:

$$G := 0 \mid x \mid [\cdot] \mid G_1|G_2 \mid (a)(G) \mid \sum_{i\in I} \pi_i.G_i \mid !\pi.G,$$

where I is a finite index set and $\pi := n(x) \mid \bar{n}m$.

Here, G is a context if all the variables in G are bound. Let F be a π-process, and $G[F]$ be the process that can use F to replace all occurrence of $[\cdot]$ in G. Also, $[\cdot]$ is left-associative and we require that $[\cdot]$ has the highest priority for convenience.

One may notice the difference between $(d)((c)(\bar{b}c.Tr_d|!c.[\![E]\!])|[\![D]\!]|!d.[\![F]\!])$ and $(c)(\bar{b}c.[\![F]\!]|!c.[\![E]\!])|[\![D]\!]$ in Example 1. If $G = (c)(\bar{b}c.[\cdot]|!c.[\![E]\!])|[\![D]\!]$, then these two processes become $(d)(G[Tr_d]|!d.[\![F]\!])$ and $G[[\![F]\!]]$ respectively, which reminds us the trigger agents in Ref. [1]. Proposition 4 given below follows the same idea in Ref. [1] and shows how to factorize $[\![F]\!]$ from $G[[\![F]\!]]$. Furthermore we will see that this kind of process pair has the expansion property that can give rise to a up-to technique for $\approx_\pi$, and this is useful to prove the soundness property of the interpretation. In order to prove Proposition 4 we need the following two auxiliary lemmas.

Lemma 4 Suppose $bn(G) \cap fn(F) = \emptyset$, and $G[F] \xrightarrow{l} P$, then there is a G' such that one of the following statements holds:

(1) $P \equiv G'[F]$ and $\forall E$ with $fn(E) \cap bn(G) = \emptyset$, $G[E] \xrightarrow{l} G'[E]$;
(2) $P \equiv G'[F]|F'$, $F \xrightarrow{l} F'$ and $\forall E$ with $fn(E) \cap bn(G) = \emptyset$, $G[E] \equiv G'[E]|E$.

Proof By induction on the depth of the derivation tree of $G[F] \xrightarrow{l} P$, consider the different structure G. Intuitively, statement (1) is the case that l is caused by G, and statement (2) is the case that l is caused by F.

Lemma 5 Suppose $bn(G) \cap fn(F) = \emptyset$, and $G[F] \xrightarrow{\tau} P$, then there is a G' such that $\forall E$ with $bn(G) \cap fn(E) = \emptyset$ one of the following statements holds:

(1) $G[E]\stackrel{\tau}{\longrightarrow}G'[E]$ and $P\equiv G'[F]$;

(2) $G[E]\equiv G'[E]|E$, $F\stackrel{\tau}{\longrightarrow}F'$ and $P\equiv G'[F]|F'$;

(3) $G[E]\equiv G'[E]|E$, and there is a G'' such that $G'[E]|F\stackrel{\tau}{\longrightarrow}G''[E]|F'$ and $P\equiv G''[F]|F'$;

(4) $G[E]\equiv G'[E]|E|E$, $G'[E]|F|F\stackrel{\tau}{\longrightarrow}G'[E]|F_1|F_2$ and $P\equiv G'[F]|F_1|F_2$.

Proof By induction on the depth of the derivation tree of $G[F]\stackrel{\tau}{\longrightarrow}P$.

In the following proposition, Tr_c has the same meaning as in Proposition 3.

Proposition 4 Let $\mathcal{R}\stackrel{\text{def}}{=}\{(\,(\tilde{d})G[F],\ (c\cup\tilde{d})(G[Tr_c]|!c.F)\,)\mid bn(G)\cap fn(F)=\emptyset$ and c is fresh$\}$ and $\mathcal{T}\stackrel{\text{def}}{=}\equiv;\mathcal{R};\equiv$ then whenever $(P,Q)\in\mathcal{T}$, we have the following statements.

(1) If $P\stackrel{\tau}{\longrightarrow}P_1$, then one of the following items holds:
  (a) $Q\stackrel{\tau}{\longrightarrow}Q_1$ with $P_1\mathcal{T}Q_1$;
  (b) $Q\stackrel{\tau}{\longrightarrow}Q_2\stackrel{\tau}{\longrightarrow}Q_1$ with $P\mathcal{T}Q_2$ and $P_1\mathcal{T}Q_1$;
  (c) $Q\stackrel{\tau}{\longrightarrow};\stackrel{\tau}{\longrightarrow}Q_2\stackrel{\tau}{\longrightarrow}Q_1$ with $P\mathcal{T}Q_2$ and $P_1\mathcal{T}Q_1$;

(2) If $Q\stackrel{\tau}{\longrightarrow}Q_1$, then either $P\mathcal{T}Q_1$ or $P\stackrel{\tau}{\longrightarrow}P_1$ with $P_1\mathcal{T}Q_1$;

(3) If $P\stackrel{l}{\longrightarrow}P_1$, then either $Q\stackrel{l}{\longrightarrow}Q_1$ with $P_1\mathcal{T}Q_1$ or $Q\stackrel{\tau}{\longrightarrow}Q_2\stackrel{l}{\longrightarrow}Q_1$ with $P\mathcal{T}Q_2$ and $P_1\mathcal{T}Q_1$;

(4) If $Q\stackrel{l}{\longrightarrow}Q_1$, then $P\stackrel{l}{\longrightarrow}P_1$ with $P_1\mathcal{T}Q_1$;

(5) $\mathcal{T}$ is codivergent.

Proof From Lemma 4 and Lemma 5.

Corollary 1 $\mathcal{T}\subseteq\approx_\pi$.

Let $P\in\mathcal{P}_\Lambda$, with the help of relation $\mathcal{T}$ and by an induction on the structure of P, we can get the following semantic correspondence between the original process and its encoding.

Proposition 5 (Operational correspondance)

1. (a) If $P\stackrel{a(\bar{b}(0))}{\longrightarrow}P'$, then $[\![P]\!]\stackrel{ab}{\longrightarrow}\equiv[\![P']\!]$;
   (b) If $P\stackrel{(\tilde{c})\bar{a}(E)}{\longrightarrow}P'$, then $[\![P]\!]\stackrel{\bar{a}(b)}{\longrightarrow}\equiv(\tilde{c})([\![P']\!]|!b.[\![E]\!])$;
   (c) If $P\stackrel{\tau}{\longrightarrow}P'$, then $[\![P]\!]\stackrel{\tau}{\longrightarrow}\mathcal{T}^{-1}[\![P']\!]$.
2. (a) If $[\![P]\!]\stackrel{ab}{\longrightarrow}P_1$, then exists P' such that $P\stackrel{\bar{a}(\bar{b}(0))}{\longrightarrow}P'$ and $P_1\equiv[\![P']\!]$;
   (b) If $[\![P]\!]\stackrel{\bar{a}(b)}{\longrightarrow}P_1$,then exist $\tilde{c},E$, and P' such that $P\stackrel{(\tilde{c})\bar{a}(E)}{\longrightarrow}P'$ and $P_1\equiv(\tilde{c})([\![P']\!]|!\bar{b}.[\![E]\!])$;
   (c) If $[\![P]\!]\stackrel{\tau}{\longrightarrow}P_1$, then $P\stackrel{\tau}{\longrightarrow}P'$ and $P_1\mathcal{T}^{-1}[\![P']\!]$.

From Proposition 4 and Proposition 5 we can get the following corollary which is useful to get the upper bound of simulation steps.

Corollary 2 (1) For all $n \geq 1$ and $P\xrightarrow{\tau^n}P'$, there exist $m(n \leq m \leq \frac{3^n-1}{2})$ and P_1 such that $[\![P]\!]\overset{\tau^m}{\Longrightarrow}P_1$ and $P_1\mathcal{T}^n[\![P']\!]$;

(2) For all $n \geq 1$ and $[\![P]\!]\xrightarrow{\tau^n}P'$, there exists $m(1 \leq m \leq n)$ and P' such that $P\xrightarrow{\tau^m}P_1$ and $[\![P_1]\!]\mathcal{T}^m P'$.

3.2 Up-to Expansion and the Fully Abstract Result

Bisimulation up-to is a widely used technique. It can effectively reduce the size of the relation needed to get a bisimulation. It works smoothly in the strong case, as it has been first introduced in Ref. [2]. However, generally this technique cannot be directly implemented in the weak case. This is the key reason we introduce a "up-to" technique for $\approx_\pi$ based on expansion. We will use it to establish the soundness property of $[\![\cdot]\!]$.

Definition 8 A binary relation $\mathcal{R}$ is an expansion on $\mathcal{P}_\pi$ if $P\mathcal{R}Q$ implies that

(1) $\mathcal{R}$ is codivergent;
(2) If $P\xrightarrow{\tau}P'$, then $P'\mathcal{R}Q$ or Q' exists such that $Q\xrightarrow{\tau}Q'$ with $P'\mathcal{R}Q'$;
(3) If $Q\xrightarrow{\tau}Q'$, then P' and P'' exists such that $P\Longrightarrow P''\xrightarrow{\tau}P'$ with $P\mathcal{R}Q''$ and $P'\mathcal{R}Q'$;
(4) If $P\xrightarrow{l}P'$, then Q' exists such that $Q\xrightarrow{l}Q'$ with $P'\mathcal{R}Q'$;
(5) If $Q\xrightarrow{l}Q'$ then P' and P'' exist such that $P\Longrightarrow P''\xrightarrow{l}P'$ with $P''\mathcal{R}Q$ and $P'\mathcal{R}Q'$.

We say P expands Q, written as $P\approx_\pi^{\triangleright}Q$, if $P\mathcal{R}Q$ for some expansion $\mathcal{R}$. From the Definition 8 we can see that the expansion relation is an asymmetric version of $\approx_\pi$. Intuitively, if $P\approx_\pi^{\triangleright}Q$ holds, then Q computes at least as fast as P. It should be clear that $\mathcal{T}^{-1}$ is an expansion.

Corollary 3 For any expansion relation $\approx_\pi^{\triangleright\,i}$, we have $\bigcup_i \approx_\pi^{\triangleright\,i} \subseteq \approx_\pi$.

In the following we will use $\approx_\pi^{\triangleright\,*}$ as an abbreviation of $\bigcup_i \approx_\pi^{\triangleright\,i}$, and $\approx_\pi^{\triangleleft\,*}$ as the reverse of $\approx_\pi^{\triangleright\,*}$.

Lemma 6 If a symmetric binary relation $\mathcal{R}$ on $\mathcal{P}$ satisfies the following properties, then $\mathcal{R} \subseteq \approx_\pi$:

(1) If $P\stackrel{l}{\longrightarrow}P_1$, then exist Q_1 and Q_2 such that $Q\Longrightarrow Q_2\stackrel{l}{\longrightarrow}Q_1$, $P_1 \approx_\pi;\mathcal{R};\approx_\pi Q_2$, and $P_1 \approx_\pi;\mathcal{R};\approx_\pi Q_1$;

(2) If $P\stackrel{\tau}{\longrightarrow}P_1$, then one of the following holds
 (a) there exist Q_1, Q_2 such that $Q\Longrightarrow Q_2\stackrel{\tau}{\longrightarrow}Q_1$, $P_1 \approx_\pi;\mathcal{R};\approx_\pi Q_2$, and $P_1\approx_\pi^{\triangleright};\mathcal{R};\approx_\pi Q_1$;
 (b) there exists Q_1 such that $Q\Longrightarrow Q_1$, $P_1\approx_\pi^{\triangleright};\mathcal{R};\approx_\pi Q_1$, and $P_1\mathcal{R};\approx_\pi Q_1$;

(3) If $P\stackrel{\tau}{\longrightarrow}P_1\stackrel{\tau}{\longrightarrow}\cdots\stackrel{\tau}{\longrightarrow}P_n\stackrel{\tau}{\longrightarrow}\cdots$ is an infinite internal action sequence, then there exist some Q' and $i \geq 1$ such that $Q\stackrel{\tau}{\Longrightarrow}Q'$ and $P_i \approx_\pi;\mathcal{R};\approx_\pi^{\triangleleft *}Q'$.

We call such $\mathcal{R}$ an external bisimulation up-to $\approx_\pi^{\triangleleft}$.

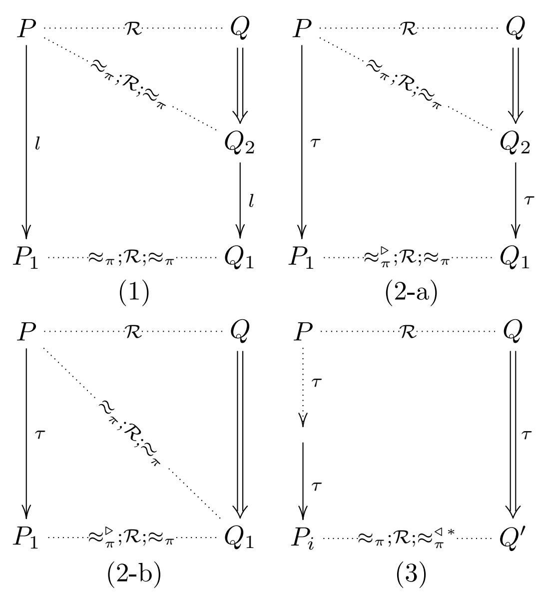

Fig. 3 Up-to expansion on $\mathcal{P}_\pi$

Proof It is sufficient to show that $\approx_\pi;\mathcal{R};\approx_\pi$ is an external bisimulation on $\mathcal{P}_\pi$. Suppose $M \approx_\pi P\mathcal{R}Q \approx_\pi N$, there are three cases to be considered:

(1) $M\stackrel{l}{\longrightarrow}M'$. There exist P' and P'' such that $P\Longrightarrow P''\stackrel{l}{\longrightarrow}P'$, $M \approx_\pi P''$, and $M' \approx_\pi P'$. The expansion property of $\approx_\pi^{\triangleright}$ makes sure that if $P\stackrel{\tau^n}{\longrightarrow}P''(n \geq 0)$, then exist $0 \leq m \leq n, Q'', s.t.\ Q\Longrightarrow Q''$ and $P''{\approx_\pi^{\triangleright}}^m\mathcal{R};\approx_\pi Q''$. Then using simple diagram-chasing, we can get N'', N' such that $N\Longrightarrow N''\stackrel{l}{\longrightarrow}N'$, $M \approx_\pi ;{\approx_\pi^{\triangleright}}^m;\approx_\pi;\mathcal{R};\approx_\pi N''$, and $M' \approx_\pi;{\approx_\pi^{\triangleright}}^m;\approx_\pi;\mathcal{R};\approx_\pi N'$. Thus by Corollary 3, $M \approx_\pi;\mathcal{R};\approx_\pi N''$ and $M' \approx_\pi;\mathcal{R};\approx_\pi N'$.

(2) $M\overset{\tau}{\longrightarrow}M'$. This case is similar as (1) but a little more annoying, so we omit it here.

(3) $M\overset{\tau}{\longrightarrow}M_1\overset{\tau}{\longrightarrow}\cdots\overset{\tau}{\longrightarrow}M_n\cdots$ is an infinite internal action sequence. By Lemma 2 there exists an infinite τ action sequence of P

$$P\overset{\tau}{\longrightarrow}P_1\overset{\tau}{\longrightarrow}P_2\overset{\tau}{\longrightarrow}\cdots\overset{\tau}{\longrightarrow}P_n\cdots, \qquad (*)$$

and some increasing function f on $\mathbb{N}$ s.t. $M_{f(i)} = P_i$. By above mentioned Case (3), there exist some $i_1 \geq 1$ s.t. $Q\overset{\tau}{\Longrightarrow}Q_1$ for some Q_1 and $P_{i_i} \approx_\pi ;\mathcal{R};Q'_1{\approx_\pi^{\lhd}}^*Q_1$. Since the τ action sequence starting from P_{i_1} is still infinite, we can use the same argument again and we can get some $i_2 > i_1$ and Q''_1 s.t. $Q'_1\overset{\tau}{\Longrightarrow}Q''_1$ and $P_{i_2} \approx_\pi;\mathcal{R};{\approx_\pi^{\lhd}}^*Q''_1$. By Definition 8, $Q_1\overset{\tau}{\Longrightarrow}Q_2$ for some Q_2 and $Q''_1{\approx_\pi^{\lhd}}^*Q_2$, thus we have $P_{i_2} \approx_\pi;\mathcal{R};{\approx_\pi^{\lhd}}^*Q_2$. Reapeatly doing this we can get an infinite τ action sequence of Q

$$Q\overset{\tau}{\Longrightarrow}Q_1\overset{\tau}{\Longrightarrow}Q_2\overset{\tau}{\Longrightarrow}\cdots\overset{\tau}{\Longrightarrow}Q_k\overset{\tau}{\Longrightarrow}\cdots$$

and an increasing index sequence $i_1 < i_2 < \cdots < i_k < \cdots$ s.t.

$$P_{i_k} \approx_\pi;\mathcal{R};{\approx_\pi^{\lhd}}^*Q_k \quad (\forall k \geq 1)$$

Then by codivergent there exist some $\hat{k}$ and N' s.t. $N\overset{\tau}{\Longrightarrow}N'$ and $Q_{\hat{k}} \approx_\pi N'$. As a result we have the following relation:

$$M_{f(i_{\hat{k}})} \approx_\pi P_{i_{\hat{k}}} \approx_\pi;\mathcal{R};{\approx_\pi^{\lhd}}^*;\approx_\pi Q_{\hat{k}} \approx_\pi N'$$

Now we are ready to prove the soundness of the interpretation.

Proposition 6 (Soundness) If $P =_\Lambda Q$, then $[\![P]\!] =_\pi [\![Q]\!]$.

Proof Let $\mathcal{R}$ be defined as follows, then by Lemma 6 we only need to prove that $\mathcal{R}$ is external bisimulation up-to $\approx_\pi^{\rhd}$:

$$\mathcal{R} = \{([\![P]\!], [\![Q]\!]) \mid P =_\Lambda Q\}$$

Suppose $([\![P]\!], [\![Q]\!]) \in \mathcal{R}$ and $[\![P]\!]\overset{\alpha}{\longrightarrow}P_1$, there are four cases:

(1) $\alpha = ab$. By Proposition 5, $P\overset{a(\bar{(b)}(0))}{\longrightarrow}P'$. This and $P =_\Lambda Q$ imply that there exists Q' and Q'' such that $Q\Longrightarrow Q''\overset{a(\bar{b}(0))}{\longrightarrow}Q'$ with $P =_\Lambda Q''$ and $P' =_\Lambda Q'$. By Corollary 2, there exists Q_3 such that $[\![Q]\!]\Longrightarrow Q_3 =_\pi [\![Q'']\!]$. Use

Proposition 5 again, we have $[\![Q'']\!] \xrightarrow{ab} Q_4 \equiv [\![Q']\!]$, thus there exist Q_1 and Q_2 s.t. $Q_3 \Longrightarrow Q_2 \xrightarrow{ab} Q_1$ with $Q_2 =_\pi [\![Q'']\!]$ and $Q_1 =_\pi Q_4$. In conclusion there exist Q_1 and Q_2 such that

$$[\![Q]\!] \Longrightarrow Q_2 \xrightarrow{ab} Q_1, [\![P]\!] \mathcal{R}; =_\pi Q_2, \text{ and } P_1 \equiv; \mathcal{R}; =_\pi Q_1$$

(2) $\alpha = \bar{a}(b)$. By Proposition 5, there exist $\widetilde{c}$, E, and P' such that $P \xrightarrow{(\widetilde{c})\bar{a}(E)} P'$ and $P_1 \equiv (\widetilde{c})([\![P']\!] | !b.[\![E]\!])$. By Theorem 1, $\forall G$ with $fn(G) \cap \{\widetilde{c}\} = \emptyset$ there exist $\widetilde{d}$, F, Q'', and Q' with $fn(G) \cap \{\tilde{d}\} = \emptyset$ such that $Q \Longrightarrow Q'' \xrightarrow{(\tilde{d})\bar{a}(F)} Q'$, $P =_\Lambda Q''$, and $(\tilde{c})(P'|G[E]) =_\Lambda (\tilde{d})(Q'|G[F])$. By Corollary 2, there exists Q_3 such that $[\![Q]\!] \Longrightarrow Q_3 =_\pi [\![Q'']\!]$. Use Proposition 5 again, there exists Q_4 such that $[\![Q'']\!] \xrightarrow{\bar{a}(b)} Q_4 \equiv (\tilde{d})([\![Q']\!] | !b.[\![F]\!])$. Let G be $!b.[\cdot]$, then $P_1 \equiv [\![(\tilde{c})(P'|G[E])]\!]$ and $Q_4 \equiv [\![(\tilde{d})(Q'|G[F])]\!]$, thus there exist Q_1 and Q_2 s.t. $Q_3 \Longrightarrow Q_2 \xrightarrow{\bar{a}(b)} Q_1$ with $Q_2 =_\pi [\![Q'']\!]$ and $Q_1 =_\pi Q_4$. In conclusion, there exist Q_1 and Q_2 such that

$$[\![Q]\!] \Longrightarrow Q_2 \xrightarrow{\bar{a}(b)} Q_1, [\![P]\!] \mathcal{R}; =_\pi Q_2, \text{ and } P_1 \equiv; \mathcal{R}; \equiv; =_\pi Q_1$$

(3) $\alpha = \tau$. By Proposition 5, $P \xrightarrow{\tau} P'$ and $P_1 \approx_\pi^{\triangleright} [\![P']\!]$. If $P \xrightarrow{\tau} P'$ is matched by $Q \Longrightarrow Q'' \xrightarrow{\tau} Q'$ for some Q' and Q'' with $P =_\Lambda Q''$ and $P =_\Lambda Q'$. By Corollary 2, there exists Q_3 such that $[\![Q]\!] \Longrightarrow Q_3 =_\pi [\![Q'']\!]$. Use Proposition 5 again, we have $[\![Q'']\!] \xrightarrow{\tau} Q_4 \approx_\pi^{\triangleright} [\![Q']\!]$ for some Q_4.

(a) If $[\![Q'']\!] \xrightarrow{\tau} Q_4$ is matched by $Q_3 \Longrightarrow Q_2 \xrightarrow{\tau} Q_1$ for some Q_2 and Q_1 with $Q_2 =_\pi [\![Q'']\!]$ and $Q_1 =_\pi Q_4$. Then we find Q_1 and Q_2, such that

$$[\![Q]\!] \Longrightarrow Q_2 \xrightarrow{\tau} Q_1, [\![P]\!] \mathcal{R}; =_\pi Q_2, \text{ and } P_1 \approx_\pi^{\triangleright}; \mathcal{R}; =_\pi Q_1$$

(b) If $[\![Q'']\!] \xrightarrow{\tau} Q_4$ is matched $Q_3 \Longrightarrow Q_1$ for some Q_1 with $[\![Q'']\!] =_\pi Q_1$ and $Q_4 =_\pi Q_1$. Then we find Q_1 such that

$$[\![Q]\!] \Longrightarrow Q_1, [\![P]\!] \mathcal{R}; =_\pi; Q_1, \text{ and } P_1 \approx_\pi^{\triangleright}; \mathcal{R}; =_\pi; Q_1$$

If $P \xrightarrow{\tau} P'$ is matched $Q \Longrightarrow Q'$ for some Q' with $P =_\Lambda Q'$ and $P' =_\Lambda Q'$, using the similar argument we can get some Q_1 such that

$$[\![Q]\!] \Longrightarrow Q_1, [\![P]\!] \mathcal{R}; =_\pi Q_1, \text{ and } P_1 \mathcal{R}; =_\pi Q_1$$

(4) Suppose $[\![P]\!] \xrightarrow{\tau} M_1 \xrightarrow{\tau} M_2 \xrightarrow{\tau} \cdots \xrightarrow{\tau} M_n \xrightarrow{\tau} \cdots$ is infinite internal action sequence, then by Corollary 2 there are $P \xrightarrow{\tau} P_1$ and $M_1 =_\pi [\![P_1]\!]$. By codi-

vergence there exist $i_1 \geq 1$ and P' such that $[\![P_1]\!] \stackrel{\tau}{\Longrightarrow} P'$ and $M_{i_1} =_\pi P'$. By Corollary 2 , we have there exists P_2 such that $P_1 \stackrel{\tau}{\Longrightarrow} P_2$ and $M_{i_1} =_\pi P' =_\pi [\![P_2]\!]$. Repeatedly doing this, we get an infinite subset indexes $\{i_1, i_2, \ldots, i_n, \ldots, \}$ and $P_1, P_2, \ldots, P_n, \ldots$ s.t. $P \stackrel{\tau}{\longrightarrow} P_1 \stackrel{\tau}{\Longrightarrow} P_2 \stackrel{\tau}{\Longrightarrow} \cdots \stackrel{\tau}{\Longrightarrow} P_n \stackrel{\tau}{\Longrightarrow} \cdots$ and $M_{i_j} =_\pi [\![P_j]\!] (j \geq 1)$. By codivergence again , there exist $k \geq 1$ and Q_1 such that $Q \stackrel{\tau}{\Longrightarrow} Q_1$ and $P_k =_\Lambda Q_1$. Use Corollary 2 again, there exists Q' such that $[\![Q]\!] \stackrel{\tau}{\Longrightarrow} Q'$ and $[\![Q_1]\!] \approx_\pi^{\triangleleft *} Q'$. In conclusion we find some $i_k \geq 1$ and Q' such that $[\![Q]\!] \stackrel{\tau}{\Longrightarrow} Q_1$ and $M_{i_k} =_\pi [\![P_k]\!] \mathcal{R} [\![Q_1]\!] \approx_\pi^{\triangleleft *} Q'$.

At last we get the main result of this section.

Theorem 3 Given $P, Q \in \mathcal{P}_\Lambda$, then $P =_\Lambda Q$ if and only if $[\![P]\!] =_\pi [\![Q]\!]$

Proof We define a relation $\mathfrak{F}$

$$\mathfrak{F} \stackrel{\text{def}}{=} \{(P, Q) \mid (P \in \mathcal{P}_\Lambda) \wedge (Q \in \mathcal{P}_\pi) \wedge ([\![P]\!] =_\pi Q)\}$$

and show that $\mathfrak{F}$ is a subbisimilarity from Λ to π. It is clear that $\mathfrak{F}$ is total and extensional. By Proposition 5, $\mathfrak{F}$ is equipollent and bisimilar. By Proposition 6, $\mathfrak{F}$ is sound. By Corollary 2 $\mathfrak{F}$ is codivergent.

4 Conclusion

In this paper, we have restudied the algebraic and expressiveness aspects of the Λ calculus in the model independent framework. We show that there is a concrete characterization of the absolute equality inside the Λ calculus. The coincidence result gives us enough reason to regard $\approx_\Lambda$ as a valid relation to the study of Λ calculus. At the same time, we show that π calculus can encode Λ calculus faithfully by proving the subbisimilarity relation. These results deepen our knowledge about Λ calculus and can be seen as one step further about the study and application of the model independent framework.

Compared to the previous work, our results have the following advantages: First we make use of the model independent framework, which is a convincing criterion and guarantees the sustainable development of the following work. Compared to the results in Ref. [6], our work is an important case study of the interaction theory. The up-to technique developed in this paper is actually a rather powerful one, especially when we want to prove the soundness property. We have found that it could be utilized in the study of encoding theory in process calculus as long as the encoding scheme satisfies some natural properties. We believe that this technique could bring us more light on the general expressiveness study. One of our future work will focus on popularizing this new technique.

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