Fundamentals of Programming Languages III

Finite and Büchi Automata

Guoqiang Li

School of Software, Shanghai Jiao Tong University

Finite Automata

Finite Automata

Finite Automaton $A = (S, \Sigma, \delta, q_0, F)$, where

- S: a finite set of states
- Σ: alphabet
- $\delta \subseteq S \times (\sigma \cup \{\varepsilon\}) \times S$: transition
- $q_0 \in S$: initial state
- $F \subseteq S$: a set of final states

Finite Automata

Finite Automaton $A = (S, \Sigma, \delta, q_0, F)$, where

- S: a finite set of states
- Σ: alphabet
- $\delta \subseteq S \times (\sigma \cup \{\varepsilon\}) \times S$: transition
- $q_0 \in S$: initial state
- $F \subseteq S$: a set of final states

Word $w = a_0 a_1 \ldots a_n \in \Sigma^*$ is accepted if there exists

$$q_0 \xrightarrow{a_1} q_1 \xrightarrow{a_2} \ldots \xrightarrow{a_n} q_n \in F$$

for $(q_{i-1}, a_i, q_i) \in \delta$

Finite Automata

Finite Automaton $A = (S, \Sigma, \delta, q_0, F)$, where

- S: a finite set of states
- Σ: alphabet
- $\delta \subseteq S \times (\sigma \cup \{\varepsilon\}) \times S$: transition
- $q_0 \in S$: initial state
- $F \subseteq S$: a set of final states

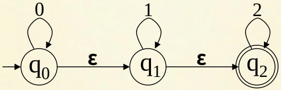
Word $w = a_0 a_1 \ldots a_n \in \Sigma^*$ is accepted if there exists

$$q_0 \xrightarrow{a_1} q_1 \xrightarrow{a_2} \ldots \xrightarrow{a_n} q_n \in F$$

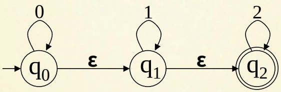
for $(q_{i-1}, a_i, q_i) \in \delta$

$L(A)$ is the set of accepted words.

Examples of Automata

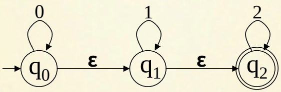


Examples of Automata

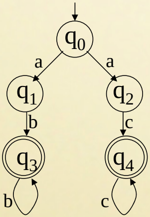


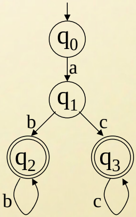
Accepts $\{0^*1^*2^*\}$

Examples of Automata

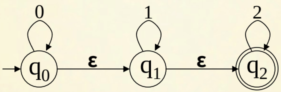


Accepts $\{0^*1^*2^*\}$

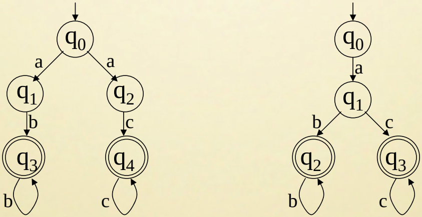




Examples of Automata



Accepts $\{0^*1^*2^*\}$



Accepts $\{ab^+, ac^+\}$

Examples of Regular Languages

$\{(ab)^n \mid \forall n \geq 0\}$

Examples of Regular Languages

$\{(ab)^n \mid \forall n \geq 0\}$

$\{(a^n b^n \mid \forall n \geq 0\}$

Examples of Regular Languages

$\{(ab)^n \mid \forall n \geq 0\}$

$\{(a^n b^n \mid \forall n \geq 0\}$

$\{ab, a^2b^2, \ldots a^n b^n\}$

Notation

$$q \xrightarrow{a} q' \Leftrightarrow (q, a, q') \in \delta$$

$$q \xrightarrow{u} q' \Leftrightarrow q = q_1 \xrightarrow{a_1} q_2 \xrightarrow{a_2} \dots \xrightarrow{a_{n-1}} q_n = q'$$

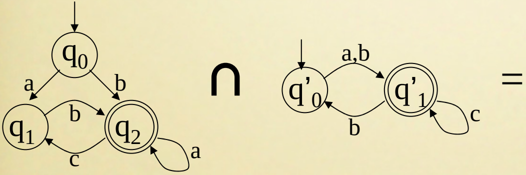
where $u = a_1 a_2 \dots a_{n-1} \in \Sigma^*$

Intersection of Automata

$A = (S, \Sigma, \delta, q_0, F), B = (S', \Sigma, \delta', q'_0, F')$

An Automaton that accepts $L(A) \cap L(B)$

$$(S \times S', \Sigma, \delta \times \delta', (q_0, q'_0), F \times F')$$

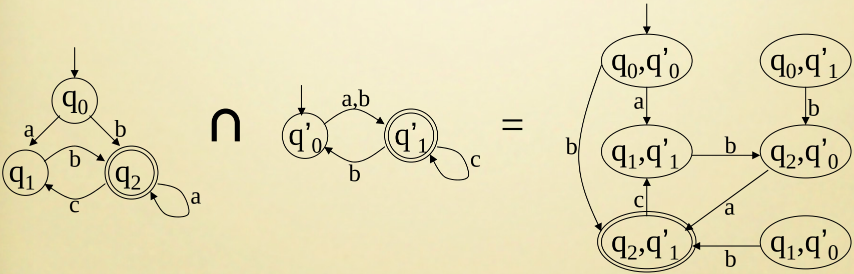


Intersection of Automata

$A = (S, \Sigma, \delta, q_0, F), B = (S', \Sigma, \delta', q_0', F')$

An Automaton that accepts $L(A) \cap L(B)$

$$(S \times S', \Sigma, \delta \times \delta', (q_0, q_0'), F \times F')$$

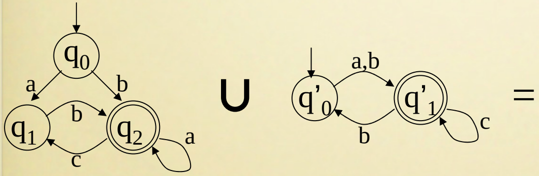


Union of Automata

$A = (S, \Sigma, \delta, q_0, F), B = (S', \Sigma, \delta', q_0', F')$

An Automaton that accepts $L(A) \cup L(B)$

$$(S \cup S' \cup \{q\}, \Sigma, \delta \cup \delta' \cup \{(q, \varepsilon, q_0), (q, \varepsilon, q_0')\}, q, F \cup F')$$

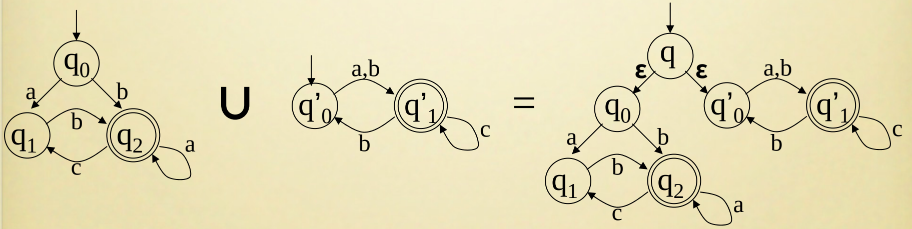


Union of Automata

$A = (S, \Sigma, \delta, q_0, F), B = (S', \Sigma, \delta', q'_0, F')$

An Automaton that accepts $L(A) \cup L(B)$

$$(S \cup S' \cup \{q\}, \Sigma, \delta \cup \delta' \cup \{(q, \varepsilon, q_0), (q, \varepsilon, q'_0)\}, q, F \cup F')$$



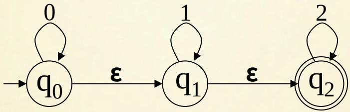
Removing ε-transitions

$A = (S, \Sigma, \delta, q_0, F)$

$$A' = (S, \Sigma, \{(q, a, q') \mid q \xrightarrow{\varepsilon^* a \varepsilon^*} q'\}, q_0, F')$$

where $F' = \begin{cases} F \cup \{q_0\} & \text{if } q_0 \xrightarrow{\varepsilon^*} q_f \text{ for } q_f \in F \\ F & \text{otherwise} \end{cases}$

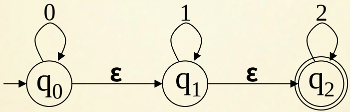
Example of ε-transition Removal



Put a new transition $\xrightarrow{a}$ where $\xrightarrow{\varepsilon^* a \varepsilon^*}$

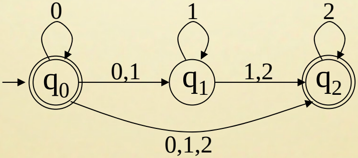
If $q_0 \xrightarrow{\varepsilon^*} q_f$ for $q_f \in F$, add q_0 to F

Example of ε-transition Removal



Put a new transition $\xrightarrow{a}$ where $\xrightarrow{\varepsilon^* a \varepsilon^*}$

If $q_0 \xrightarrow{\varepsilon^*} q_f$ for $q_f \in F$, add q_0 to F



Complement of Automata

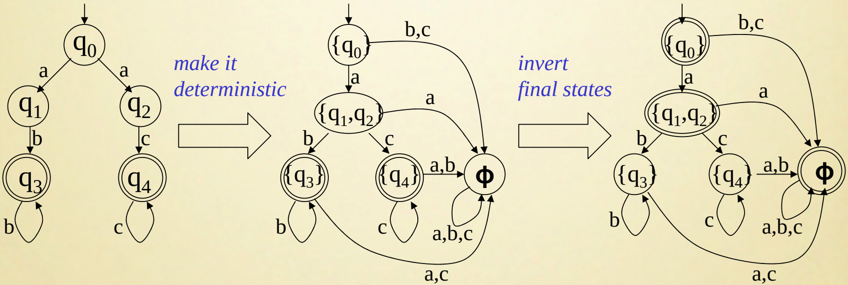
$A = (S, \Sigma, \delta, q_0, F)$

- if A is deterministic, $A^c = (S, \Sigma, \delta, q_0, S - F)$.
- if A is non-deterministic, make A deterministic first

  Assume that A is without ε-transition. Then

  $(P(S), \Sigma, \{(X, a, \{y \mid x \xrightarrow{a} y \text{ for } x \in X\})\}, \{q_0\}, \{X \mid X \cap F \neq \emptyset\})$

Example of Complement



Emptiness

Emptiness

Pumping Lemma

Let $A = (S, \Sigma, \delta, q_0, F)$ be a finite automaton. For each $z \in L(A)$ with $|z| \geq |S|$, $\exists u, v, w$ such that $z = uvw$, $|uw| < |S|$, $|v| \geq 1, uv^i w \in L(A)$.

Emptiness

Pumping Lemma

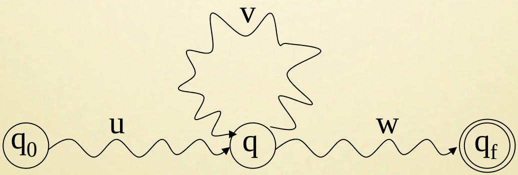
Let $A = (S, \Sigma, \delta, q_0, F)$ be a finite automaton. For each $z \in L(A)$ with $|z| \geq |S|$, $\exists u, v, w$ such that $z = uvw$, $|uw| < |S|$, $|v| \geq 1, uv^i w \in L(A)$.

$L(A) \neq \emptyset$ iff $\exists z$ with $|z| < |S|$ and $z \in L$.

Idea of Pumping Lemma

Pumping Lemma

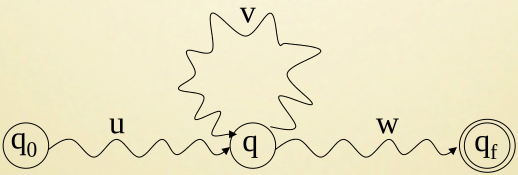
Let $A = (S, \Sigma, \delta, q_0, F)$ be a finite automaton. For each $z \in L(A)$ with $|z| \geq |S|$, $\exists u, v, w$ such that $z = uvw$, $|uw| < |S|$, $|v| \geq 1$, $uv^i w \in L(A)$.



Idea of Pumping Lemma

Pumping Lemma

Let $A = (S, \Sigma, \delta, q_0, F)$ be a finite automaton. For each $z \in L(A)$ with $|z| \geq |S|$, $\exists u, v, w$ such that $z = uvw$, $|uw| < |S|$, $|v| \geq 1$, $uv^iw \in L(A)$.



Pigeon hole principle!

Subset

$A = (S, \Sigma, \delta, q_0, F), B = (S', \Sigma, \delta', q_0', F')$

Ask $L(A) \subseteq L(B)$?

Subset

$A = (S, \Sigma, \delta, q_0, F), B = (S', \Sigma, \delta', q_0', F')$

Ask $L(A) \subseteq L(B)$?

$$L(A) \subseteq L(B) \Leftrightarrow L(A) \cap L(B^c) = \emptyset$$

Congruence

$u \, R \, v$ is a congruence iff R is an equivalence and preserved under concatenation

$$u \, R \, v \Rightarrow wuw' \, R \, wvw' \text{ for each } w, w' \in \Sigma^*$$

.

Myhill-Nerode Theorem

Myhill-Nerode Theorem

The following three statements are equivalent.

1. L is regular.
2. L is a union of congruence classes of finite index.
3. R_L is a congruence of finite index, where

$$u\, R_L\, v \text{ iff } uw \in L \Leftrightarrow vw \in L \text{ for each } w \in \Sigma^*$$

Proof: $3 \Rightarrow 1$

Let R_L be a congruence of finite index, where

$$u\,R_L\,v \text{ iff } uw \in L \Leftrightarrow vw \in L \text{ for each } w \in \Sigma^*$$

Proof: $3 \Rightarrow 1$

Let R_L be a congruence of finite index, where

$$u\, R_L\, v \text{ iff } uw \in L \Leftrightarrow vw \in L \text{ for each } w \in \Sigma^*$$

Let an automaton $A = (S, \Sigma, \delta, q_0, F)$ be

- $S = \Sigma^*/R_L$ (finite congruence classes of R_L)
- $\delta = \{([u], a, [ua]) \mid u \in \Sigma^*, a \in \Sigma\}$
- $q_0 = [\varepsilon]$
- $F = \{[u] \mid u \in L\}$

Proof: $3 \Rightarrow 1$

Let R_L be a congruence of finite index, where

$$u \, R_L \, v \text{ iff } uw \in L \Leftrightarrow vw \in L \text{ for each } w \in \Sigma^*$$

Let an automaton $A = (S, \Sigma, \delta, q_0, F)$ be

- $S = \Sigma^*/R_L$ (finite congruence classes of R_L)
- $\delta = \{([u], a, [ua]) \mid u \in \Sigma^*, a \in \Sigma\}$
- $q_0 = [\varepsilon]$
- $F = \{[u] \mid u \in L\}$

$L = L(A)$ and L is regular.

Proof: $1 \Rightarrow 2$

Let $L = L(A)$ with $A = (S, \Sigma, \delta, q_0, F)$

Proof: $1 \Rightarrow 2$

Let $L = L(A)$ with $A = (S, \Sigma, \delta, q_0, F)$

$u \, R_A \, v$ iff $q \xrightarrow{u} q' \Leftrightarrow q \xrightarrow{v} q'$ for $q, q' \in S$

Proof: $1 \Rightarrow 2$

Let $L = L(A)$ with $A = (S, \Sigma, \delta, q_0, F)$

$u \, R_A \, v$ iff $q \xrightarrow{u} q' \Leftrightarrow q \xrightarrow{v} q'$ for $q, q' \in S$

R_A is a congruece of finite index,

Proof: $1 \Rightarrow 2$

Let $L = L(A)$ with $A = (S, \Sigma, \delta, q_0, F)$

$u \, R_A \, v$ iff $q \xrightarrow{u} q' \Leftrightarrow q \xrightarrow{v} q'$ for $q, q' \in S$

R_A is a congruece of finite index, (at most $2^{|S| \times |S|}$).

Proof: $2 \Rightarrow 3$

Let R be a congruence of finite index and let L be a union of congruence classes.

Proof: $2 \Rightarrow 3$

Let R be a congruence of finite index and let L be a union of congruence classes.

Let $u\, R_L\, v$ iff $uw \in L \Leftrightarrow vw \in L$ for each $w \in \Sigma^*$.

Proof: $2 \Rightarrow 3$

Let R be a congruence of finite index and let L be a union of congruence classes.

Let $u \, R_L \, v$ iff $uw \in L \Leftrightarrow vw \in L$ for each $w \in \Sigma^*$.

$u \, R \, v \Rightarrow u \, R_L \, v$; thus, R_L is of finite index.

Another Technique for Complement

Myhill-Nerode Theorem says that L is regular $\Leftrightarrow$ L is a union of congruence classes of finite index.

Another Technique for Complement

Myhill-Nerode Theorem says that L is regular $\Leftrightarrow$ L is a union of congruence classes of finite index.

$$L = \bigcup_{U_i \cap L \neq \emptyset} U_i$$

Another Technique for Complement

Myhill-Nerode Theorem says that L is regular $\Leftrightarrow$ L is a union of congruence classes of finite index.

$$L = \bigcup_{U_i \cap L \neq \emptyset} U_i$$

Note that each U_i is regular! Thus,

$$L^c = \bigcup_{U_i \cap L = \emptyset} U_i$$

Büchi Automata

Büchi Automata

Accept ω-words, instead of finite words.

Acceptance condition is different.

Büchi Automata

Accept ω-words, instead of finite words.

Acceptance condition is different.

For a Büchi automaton $A = (S, \Sigma, \delta, q_0, F)$, ω-word $w = a_0a_1 \ldots \in \Sigma^\omega$ is accepted if there exists $q_0 \xrightarrow{a_0} q_1 \xrightarrow{a_1} \ldots$ for $(q_i, a_i, q_{i+1}) \in \delta$ across F infinitely often. $L(A)$ is the set of accepted ω-words.

Büchi Automata

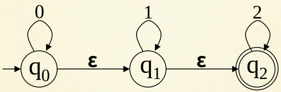
Accept ω-words, instead of finite words.

Acceptance condition is different.

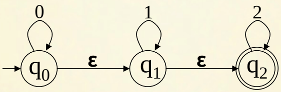
For a Büchi automaton $A = (S, \Sigma, \delta, q_0, F)$, ω-word $w = a_0 a_1 \ldots \in \Sigma^\omega$ is accepted if there exists $q_0 \xrightarrow{a_0} q_1 \xrightarrow{a_1} \ldots$ for $(q_i, a_i, q_{i+1}) \in \delta$ across F infinitely often. $L(A)$ is the set of accepted ω-words.

$L(A)$ is called regular (ω-language).

Examples of Büchi Automata

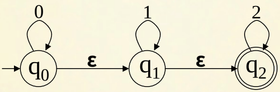


Examples of Büchi Automata

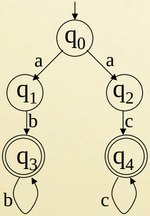


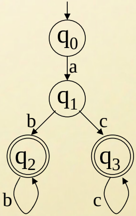
Accepts $\{0^*1^*2^\omega\}$

Examples of Büchi Automata

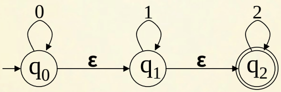


Accepts $\{0^*1^*2^\omega\}$

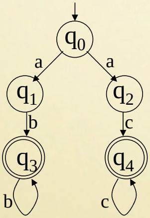


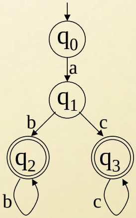


Examples of Büchi Automata



Accepts $\{0^*1^*2^\omega\}$





Accepts $\{ab^\omega, ac^\omega\}$

Deterministic VS. Non-deterministic

Accepted by deterministic Büchi automata?

- $(b^*a)^\omega$

- $(b^*a)^*b^\omega$

Deterministic VS. Non-deterministic

Accepted by deterministic Büchi automata?

- $(b^*a)^\omega$
    - a appears infinitely many
    - Yes!
- $(b^*a)^*b^\omega$

Deterministic VS. Non-deterministic

Accepted by deterministic Büchi automata?

- $(b^*a)^\omega$
  - a appears infinitely many
  - Yes!
- $(b^*a)^*b^\omega$
  - $\{a, b\}^\omega - (b^*a)^\omega$

Deterministic VS. Non-deterministic

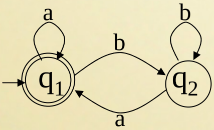
Accepted by deterministic Büchi automata?

- $(b^*a)^\omega$
  - a appears infinitely many
  - Yes!
- $(b^*a)^*b^\omega$
  - $\{a,b\}^\omega - (b^*a)^\omega$
  - No!

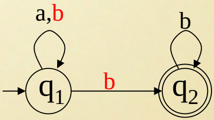
Deterministic VS. Non-deterministic

Accepted by deterministic Büchi automata?

- $(b^*a)^\omega$
  - a appears infinitely many
  - Yes!
- $(b^*a)^*b^\omega$
  - $\{a, b\}^\omega - (b^*a)^\omega$
  - No!



$(b^*a)^\omega$



$(b^*a)^*b^\omega$

Variation on Automata for ω-Words

Let $Inf(\sigma)$ be the set of states that a path σ across infinitely often. Let $\alpha \in \Sigma^\omega$

Variation on Automata for ω-Words

Let $\mathit{Inf}(\sigma)$ be the set of states that a path σ across infinitely often. Let $\alpha \in \Sigma^\omega$

Büchi automata $A = (S, \Sigma, \delta, q_0, F)$

- α has a path σ such that $\mathit{Inf}(\sigma) \cap F \neq \emptyset$.

Muller automata $A = (S, \Sigma, \delta, q_0, \{F_1, \ldots, F_m\})$

- α has a path σ such that $\mathit{Inf}(\sigma) = F_i$.

Rabin automata $A = (S, \Sigma, \delta, q_0, \{(L_1, M_1), \ldots, (L_m, M_m)\})$

- α has a path σ such that $\mathit{Inf}(\sigma) \cap L_i = \emptyset, \mathit{Inf}(\sigma) \cap M_i \neq \emptyset$.

Variation on Automata for ω-Words

Let $Inf(\sigma)$ be the set of states that a path σ across infinitely often. Let $\alpha \in \Sigma^\omega$

Büchi automata $A = (S, \Sigma, \delta, q_0, F)$

- α has a path σ such that $Inf(\sigma) \cap F \neq \emptyset$.

Muller automata $A = (S, \Sigma, \delta, q_0, \{F_1, \ldots, F_m\})$

- α has a path σ such that $Inf(\sigma) = F_i$.

Rabin automata $A = (S, \Sigma, \delta, q_0, \{(L_1, M_1), \ldots, (L_m, M_m)\})$

- α has a path σ such that $Inf(\sigma) \cap L_i = \emptyset, Inf(\sigma) \cap M_i \neq \emptyset$.

If nondeterministic, they are all equivalent.

Algorithm Reuse

Q: Which algorithms of finite automata can be reused in Büchi automata?

Algorithm Reuse

Q: Which algorithms of finite automata can be reused in Büchi automata?

- Intersection?
- Union?
- Complement?
- Emptiness?
- Subset?

Boolean Closure of Büchi Automata

Boolean Closure of Büchi Automata

$A = (S, \Sigma, \delta, q_0, F), B = (S', \Sigma, \delta', q'_0, F'),$

Boolean Closure of Büchi Automata

$A = (S, \Sigma, \delta, q_0, F), B = (S', \Sigma, \delta', q_0', F'),$

Büchi automaton that accepts $L(A) \cap L(B)$
$(S \times S' \times \{0,1\}, \Sigma, \delta'', (q_0, q_0', 0), F \times S' \times \{0\} \cup S \times F' \times \{1\})$

where $\delta'' = \{((s, s', i), (a, a'), (t, t', j)) \mid (s, a, t) \in \delta, (s', a', t') \in \delta',$

- $j = 1$ if either $i = 0$ and $t \in F$, or $i = 1$ and $t' \notin F'$,
- $j = 0$ if either $i = 0$ and $t \notin F$, or $i = 1$ and $t' \in F'$

$\}$

Boolean Closure of Büchi Automata

$A = (S, \Sigma, \delta, q_0, F), B = (S', \Sigma, \delta', q_0', F'),$

Büchi automaton that accepts $L(A) \cap L(B)$
$(S \times S' \times \{0, 1\}, \Sigma, \delta'', (q_0, q_0', 0), F \times S' \times \{0\} \cup S \times F' \times \{1\})$

where $\delta'' = \{((s, s', i), (a, a'), (t, t', j)) \mid (s, a, t) \in \delta, (s', a', t') \in \delta',$

- $j = 1$ if either $i = 0$ and $t \in F$, or $i = 1$ and $t' \notin F'$,
- $j = 0$ if either $i = 0$ and $t \notin F$, or $i = 1$ and $t' \in F'$

$\}$

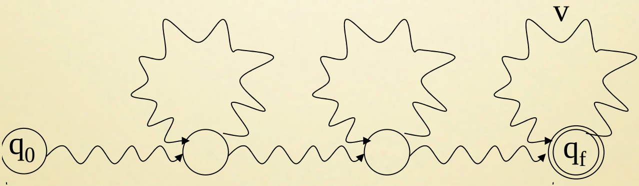
Büchi automaton that accepts $L(A) \cup L(B)$
$(S \cup S' \cup \{q\}, \Sigma, \delta \cup \delta' \cup \{(q, \varepsilon, q_0), (q, \varepsilon, q_0')\}, q, F \cup F')$

Emptiness

For a Büchi automaton $A = (S, \Sigma, \delta, q_0, F)$, $L(A) \neq \emptyset$ implies $\exists u, v \in \Sigma^*$ such that $|u|, |v| \leq |S|$ and $uv^\omega \in L(A)$.

Emptiness

For a Büchi automaton $A = (S, \Sigma, \delta, q_0, F)$, $L(A) \neq \emptyset$ implies $\exists u, v \in \Sigma^*$ such that $|u|, |v| \leq |S|$ and $uv^\omega \in L(A)$.



Complement of Büchi Automata

For a Büchi automaton $A = (S, \Sigma, \delta, q_0, F)$,

Complement of Büchi Automata

For a Büchi automaton $A = (S, \Sigma, \delta, q_0, F)$,

Even if A is deterministic, A^c may be non-deterministic.

- $(b^*a)^\omega$

Complement of Büchi Automata

For a Büchi automaton $A = (S, \Sigma, \delta, q_0, F)$,

Even if A is deterministic, A^c may be non-deterministic.

- $(b^*a)^\omega$

But, deterministic $\subset$ non-deterministic …

Complement of Büchi Automata

For a Büchi automaton $A = (S, \Sigma, \delta, q_0, F)$,

Even if A is deterministic, A^c may be non-deterministic.

- $(b^*a)^\omega$

But, deterministic $\subset$ non-deterministic …

The Büchi automaton version of Myhill-Nerode Theorem discussion is required.

Several ways for Complement

Several ways for Complement

Via Muller Automata

Several ways for Complement

Via Muller Automata

Via Alternating Automata

Miyano, S., Hayashi, T., Alternating Finite Automata on omega-Words. TCS 32, pp.321-330, 1984

Several ways for Complement

Via Muller Automata

Via Alternating Automata

Miyano, S., Hayashi, T., Alternating Finite Automata on omega-Words. TCS 32, pp.321-330, 1984

Explicit representation by congruence classes (this also gives minimization)

Many many new techniques recently, which is still a hot topic nowadays.

Notations

For a Büchi automaton $A = (S, \Sigma, \delta, q_0, F)$,

$q \xrightarrow{u}_F q' \Leftrightarrow q \xrightarrow{u} q'$ across some $q_f \in F$

$u \sim_A v$ iff $\forall q, q' \in S$,

$$(q \xrightarrow{u} q' \Leftrightarrow q \xrightarrow{v} q') \wedge (q \xrightarrow{u}_F q' \Leftrightarrow q \xrightarrow{v}_F q')$$

Finite Congruence of Büchi Automata

$\sim_A$ is a finite congruence over Σ

$\sim_A$ classes U, V are regular

$U.V^\omega$ is regular ω-languages.

$L(A)$ as a Union of $U.V^\omega$

Lemma

For a Büchi automaton $A = (S, \Sigma, \delta, q_0, F)$,

$$L(A) = \bigcup_{U.V^\omega \cap L(A) \neq \emptyset} U.V^\omega$$

$L(A)$ as a Union of $U.V^\omega$

Lemma

For a Büchi automaton $A = (S, \Sigma, \delta, q_0, F)$,

$$L(A) = \bigcup_{U.V^\omega \cap L(A) \neq \emptyset} U.V^\omega$$

$L(A) \supseteq \bigcup_{U.V^\omega \cap L(A) \neq \emptyset} U.V^\omega$ is easy, since

$$U.V^\omega \cap L(A) \neq \emptyset \Rightarrow U.V^\omega \subseteq L(A)$$

$L(A)$ as a Union of $U.V^\omega$

Lemma

For a Büchi automaton $A = (S, \Sigma, \delta, q_0, F)$,

$$L(A) = \bigcup_{U.V^\omega \cap L(A) \neq \emptyset} U.V^\omega$$

$L(A) \supseteq \bigcup_{U.V^\omega \cap L(A) \neq \emptyset} U.V^\omega$ is easy, since

$$U.V^\omega \cap L(A) \neq \emptyset \Rightarrow U.V^\omega \subseteq L(A)$$

$L(A) \subseteq \bigcup_{U.V^\omega \cap L(A) \neq \emptyset} U.V^\omega$ needs Ramsey's Theorem.

Ramsey's Theorem

General Version

Let G be an infinite complete graph. Put a label from $\{1, 2, \ldots, k\}$ on each edge. Then, there exists an infinite complete sub-graph such that its each edge has the same label.

$L(A) \subseteq \bigcup_{U.V^\omega \cap L(A) \neq \emptyset} U.V^\omega$

For each $\alpha \in \Sigma^\omega$, there exist $\sim_A$ -classes U, V such that $\alpha \in U.V^\omega$.

Let $\alpha(m,n) = a_m a_{m+1} \ldots a_{n-1}$ for $\alpha = a_1 a_2 a_3 \ldots$

Regarding $\alpha(m,n) \in V$ as a label V, by Ramsey's Theorem, $\alpha(n_1, n_2), (n_2, n_3), \ldots \in V$. If $\alpha(0, n_1) \in U$, then $\alpha \in U.V^\omega$.

Complement of $L(A)$

For a Büchi automaton $A = (S, \Sigma, \delta, q_0, F)$,

$$L(A) = \bigcup_{U.V^\omega \cap L(A) \neq \emptyset} U.V^\omega$$

Then,

$$L^c(A) = \bigcup_{U.V^\omega \cap L(A) = \emptyset} U.V^\omega$$

U, V are regular; thus, $L^c(A)$ is regular.

Reports

Rep4. Antichain for Universality, subset of automata (Maximal 3 students).