

# Fundamentals of Programming Languages IX

Conclusion

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# Report

The deadline is firmly on Jan. 10, 2018!

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Both paper version and electronic version are needed ( **3203 Software Building** & [li.g@outlook.com](mailto:li.g@outlook.com))!

# Scoring Policy

- 10% Attendance.
- 20% Homework.
  - Four assignments.
  - Each one is 5pts.
  - Work out individually.
  - Each assignment will be evaluated by *A, B, C, D, F* (**Excellent(5)**, **Good(5)**, **Fair(4)**, **Delay(3)**, **Fail(0)**)
- 70% Final report.

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Homework is MUCH MORE IMPORTANT than report!

# What We Have Learnt

- Theoretical computer sciences
- Program analysis (program analysis = abstraction interpretation + model checking)
- Program semantics
- Functional program language basics

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  - **propositional logics** (preliminary)
  - **set theory** (program semantics)
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  - **lambda calculus**
  - **simple type of lambda calculus**

# Theoretical Computer Sciences

# Propositional Logic

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Syntax and semantics

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Satisfiability and validity

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Normal form

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- **Tseitin's Encoding**

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Proof system

- Natural deduction
- Sequent calculus

# Set Theory

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Lambda notation, composition of functions, identity function, inverse,  
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**Georg Cantor's Diagonal Argument**

# Abstract Algebra

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## Preorder and partial order

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- **partial order**: reflexivity, transitivity, antisymmetry

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Tarski's fixpoint theorem

# Automata Theory

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Finite automata: DFA, NFA

Infinite automata: Büchi automata

Pushdown automata and pushdown systems

Computation on automata

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Mathematical induction

Course-of-values induction

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**Structural induction**

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**Definition by induction**

# Program Analysis

# Slogan

Program Analysis = Abstract Interpretation + Model Checking

# Abstraction Interpretation

Specify an analysis in terms of the following data:

- 1 **Preset**  $D$  (abstract domain).
- 2 **Monotone function**  $F$  (abstract transfer function).
- 3 **Widening** operator  $\nabla$  (unless  $D$  is finite).
- 4 **Narrowing** operator  $\Delta$  (optional).
- 5 **Galois connection** from  $C$  (concrete domain) to  $D$ .
- 6 **Soundness** of  $F$  wrt.  $E$  (concrete transfer function).

Then, the analysis terminates and is correct (sound).

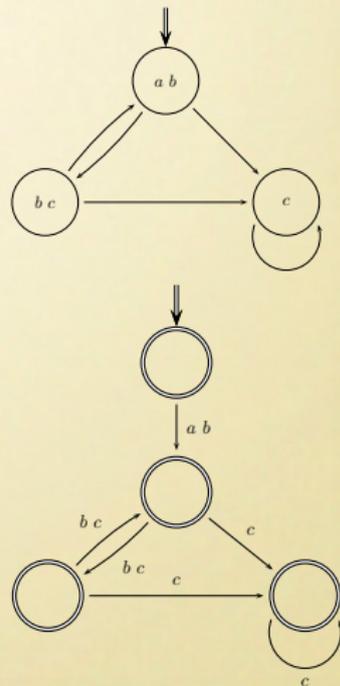
# Abstraction Interpretation

## Patrick Cousot awarded John von Neumann Medal

[Patrick Cousot](#) is the recipient of the [IEEE John von Neumann medal](#), given "for outstanding achievements in computer-related science and technology". The medal citation states that he is being recognized "for introducing abstract interpretation, a powerful framework for automatically calculating program properties with broad application to verification and optimization." Congratulations!

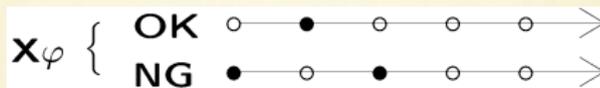
# Model Checking: Model

- Kripke structure:  $M = (S, S_0, R, L)$ 
  - $S$ , finite set of state
  - $S_0 \subseteq S$ , initial state
  - $R \subseteq S \times S$ , transition relations
  - $L : S \rightarrow 2^{AP}$ , status label function  
( $AP$ : atomic propositions)
- Finite automata:  $\mathcal{A} = (\Sigma, Q, Q_0, F, \delta)$ 
  - $A$ , finite set of input alphabet
  - $Q$ , finite set of control location
  - $Q_0 \subseteq Q$ , initial control locations
  - $F \subseteq Q$ , final control locations
  - $\delta \subseteq Q \times \Sigma \times Q$ , transitions

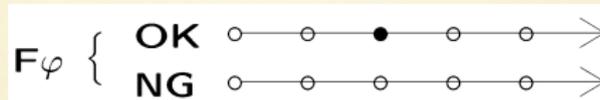


# Model Checking: Temporal Logic

- Next



- Finally



- Globally



- Until



# CTL Vs. LTL

- CTL: temporal operators must be immediately followed by path quantifiers.
  - e.g.,  $AF\varphi$ ,  $EG\varphi$ ,  $AXEG\varphi$ ,  $EXA(\varphi U\psi)$
- LTL: path quantifiers are allowed only at the outermost position.
  - e.g.,  $AGF\varphi$ ,  $EX(\varphi U\psi)$ ,  $A(F\varphi \vee G\psi)$
- Except for fairness, most properties are expressed in  $CTL \cap LTL$ .

# Decidability of CTL and LTL

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- by tableau
- by **Büchi automata**: on-the-fly model checking
- by **SAT**: bounded model checking

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Timed automata, well-structured transition systems (WSTS)

# Program Semantics

operational semantics

denotational semantics

axiomatic semantics

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Hoare logic, separation logic

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Domain theory

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Gödel incompleteness

# Functional Programming Languages

# Lambda Calculus and Types

Lambda calculus syntax

$\beta$  reduction and  $\eta$  reduction

Full  $\beta$ -reduction, normal order strategy, call by name, call by value

Programming in Lambda calculus

Church-Rosser Property

De Bruijn representation of Lambda calculus

Simple types of Lambda calculus

Progress and preservation

# What We Did Not and Should Learn?

How to program in functional programming languages, such as SML, Ocaml, Haskell

Algorithms in functional programming languages

Structured types

Type checking and inferences

Normalization, References and Exceptions

Subtyping, recursive types

Higher order systems