Homework#4 Divide and Conquer

Textbook:

6.35. Let $A[1..n]$ be a set of integers. Give an algorithm to reorder the elements in A so that all negative integers are positioned to the left of all nonnegative integers. Your algorithm should run in time $\Theta(n)$.

6.2. Consider Algorithm SLOWMINMAX which is obtained from Algorithm MINMAX by replacing the test

$$\textbf{if } high - low = 1$$

by the test

$$\textbf{if } high = low$$

and making some other changes in the algorithm accordingly. Thus, in Algorithm SLOWMINMAX, the recursion is halted when the size of the input array is 1. Count the number of comparisons required by this algorithm to find the minimum and maximum of an array $A[1..n]$, where n is a power of 2. Explain why the number of comparisons in this algorithm is greater than that in Algorithm MINMAX. (Hint: In this case, the initial condition is $C(1) = 0$).

6.52. Give a divide-and-conquer algorithm to find the second largest element in an array of n numbers. Derive the time complexity of your algorithm.

Counting inversions. An *inversion* in an array *a*[] is a pair of entries *a*[*i*] and *a*[*j*] such that *i*<*j* but *a*[*i*]>*a*[*j*]. Given an array, design a linearithmic (O(n*log*n)) algorithm to count the number of inversions.

Space complexity of Quicksort. Modify QUICKSORT to ensure its work space is Θ(log n).

Nuts and bolts. A disorganized carpenter has a mixed pile of *N* nuts and *N* bolts. The goal is to find the corresponding pairs of nuts and bolts. Each nut fits exactly one bolt and each bolt fits exactly one nut. By fitting a nut and a bolt together, the carpenter can see which one is bigger (but the carpenter cannot compare two nuts or two bolts directly). Design an algorithm for the problem that uses *N*log*N* compares (probabilistically).

Oil pipeline. Professor Olay is consulting for an oil company, which is planning a large pipeline running east to west through an oil field of n wells. The company wants to connect a spur pipeline from each well directly to the main pipeline along a shortest route (either north or south), as shown in the following Figure. Given the x- and y-coordinates of the wells, how should the professor pick the optimal location of the main pipeline, which would be the one that minimizes the total length of the spurs? Show how to determine the optimal location in linear time.

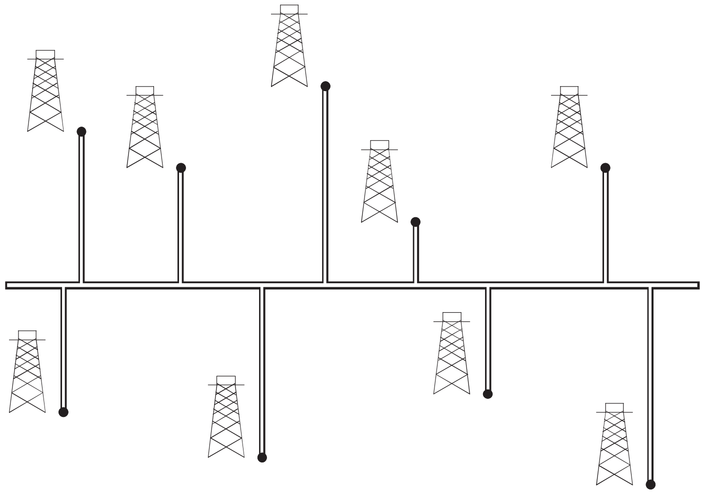

Fig. Professor Olay needs to determine the position of the east-west oil pipeline that minimizes the total length of the north-south spurs.