Homework#7 Shortest Path

Textbook:

8.16. Modify Algorithm DIJKSTRA so that it finds the shortest paths in addition to their lengths.

8.19. Give an example of a directed graph to show that Algorithm DIJKSTRA does not always work if some of the edges have negative weights.

TrueOrFalse. (If the statement is true, prove it; otherwise, give a counter example.)

a) Let P be a shortest path from some s to t in an edge-weighted digraph G. If the weight of each edge in G is increased by one, then P will still be a shortest path from s to t in the modified digraph G'.

b) If you run Dijkstra's algorithm on an edge-weighted DAG with positive weights, the order in which the vertices are picked in step "Find the vertex that has the minimum λ" is a topological order.

c) Let G be a directed graph with positive edge weights. Suppose that you increase the length of an edge by x. Then, the length of the shortest path from s to t can increase by more than x.

d) Bellman-Ford finds the shortest simple path from s to every other vertex, even if the edge weights are positive or negative integers, provided there are no negative cycles.

Paths in DAG. Give an efficient algorithm to count the total number of paths in a directed acyclic graph. Analyze your algorithm.

Shortest path tree. You are given a directed graph G = (V, E) with (possibly negative) weighted edges, along with a specific node s ∈ V and a tree T = (V, E′), E′ □ E. Give an algorithm that checks whether T is a shortest-path tree for G with starting point s. Your algorithm should run in linear time.