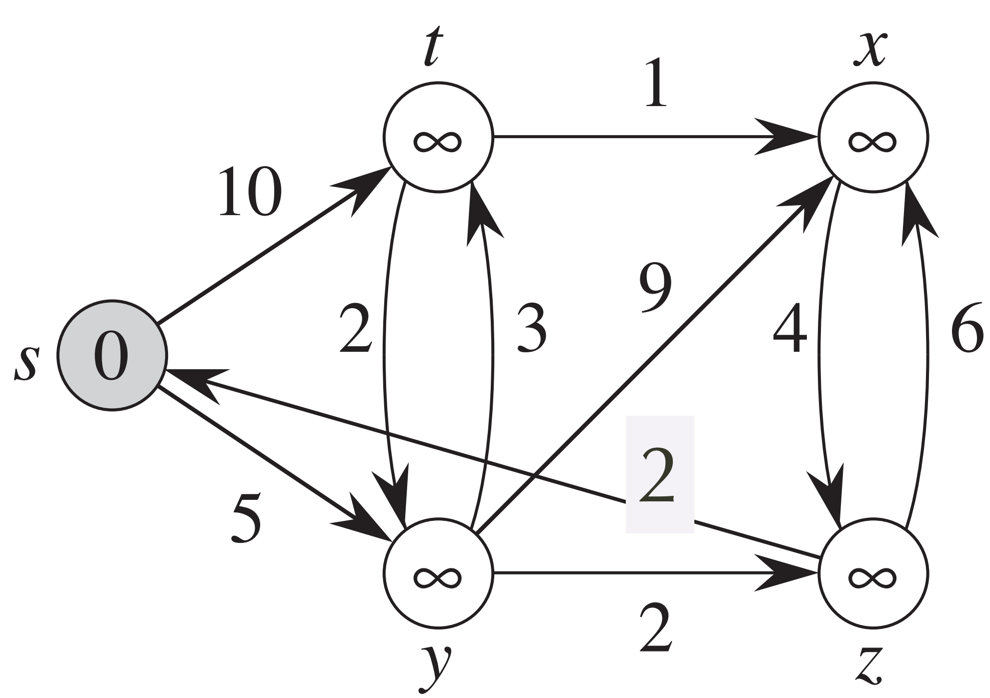
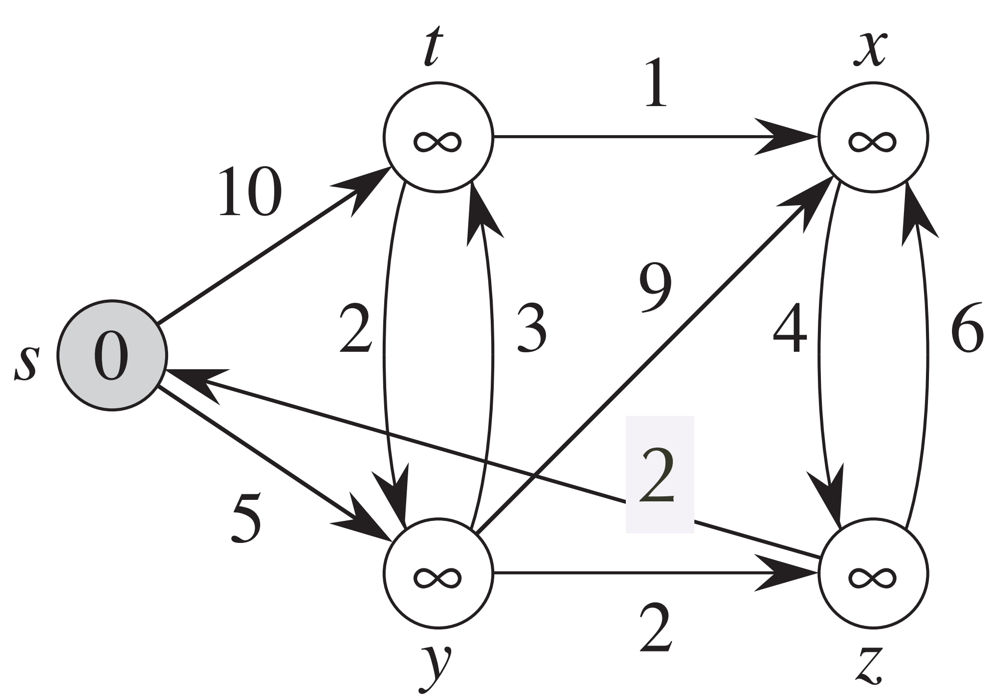
Floyd-Warshall algorithm



$$\begin{array}{c|ccccc} & s & t & x & y & z \\ \hline s & 0 & 10 & \infty & 5 & \infty \\ t & \infty & 0 & 1 & 2 & \infty \\ x & \infty & \infty & 0 & \infty & 4 \\ y & \infty & 3 & 9 & 0 & 2 \\ z & 2 & \infty & 6 & \infty & 0 \end{array}$$

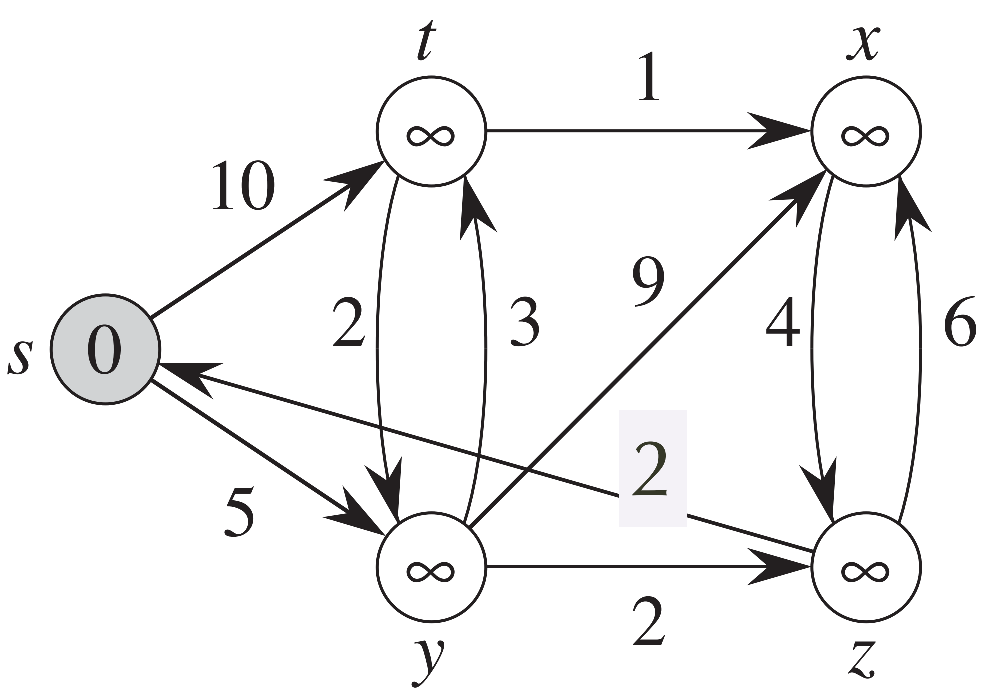
Floyd-Warshall algorithm



$$
\begin{array}{c c}
 & \begin{array}{ccccc} s & t & x & y & z \end{array} \\
\begin{array}{c} s \\ t \\ x \\ y \\ z \end{array} &
\begin{bmatrix}
0 & 10 & \infty & 5 & \infty \\
\infty & 0 & 1 & 2 & \infty \\
\infty & \infty & 0 & \infty & 4 \\
\infty & 3 & 9 & 0 & 2 \\
2 & \infty & 6 & \infty & 0
\end{bmatrix}
\end{array}
$$

The shortest path from u to v that passes none vertex

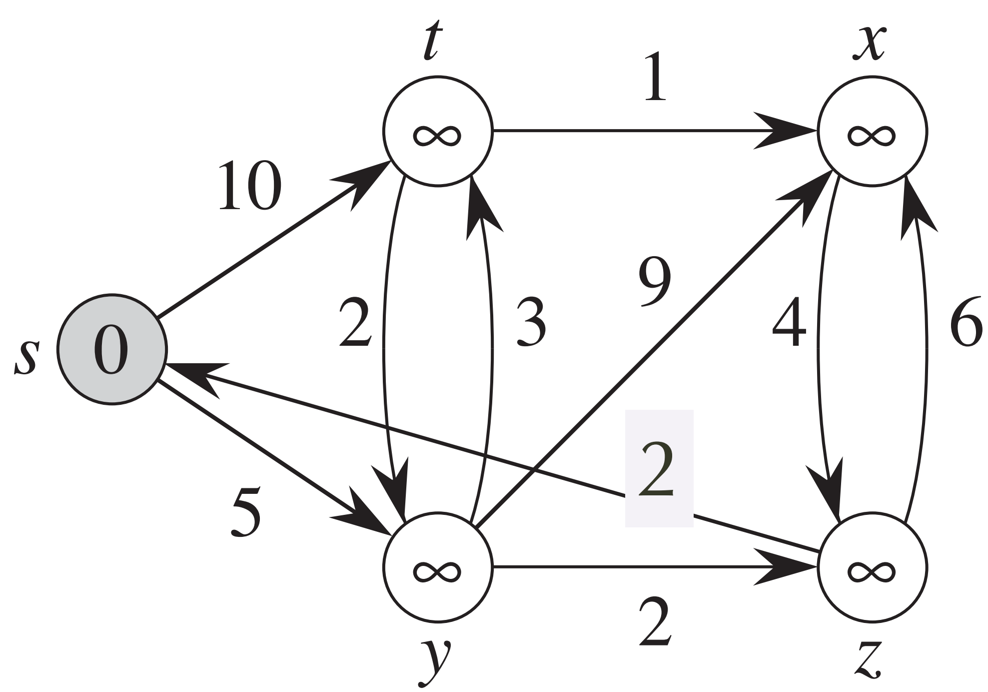
Floyd-Warshall algorithm



$$
\begin{array}{c|c}
 & \begin{array}{ccccc} s & t & x & y & z \end{array} \\
\begin{array}{c} s \\ t \\ x \\ y \\ z \end{array} &
\begin{bmatrix}
0 & 10 & \infty & 5 & \infty \\
\infty & 0 & 1 & 2 & \infty \\
\infty & \infty & 0 & \infty & 4 \\
\infty & 3 & 9 & 0 & 2 \\
2 & \infty & 6 & \infty & 0
\end{bmatrix}
\end{array}
$$

The shortest path from u to v that passes none vertex

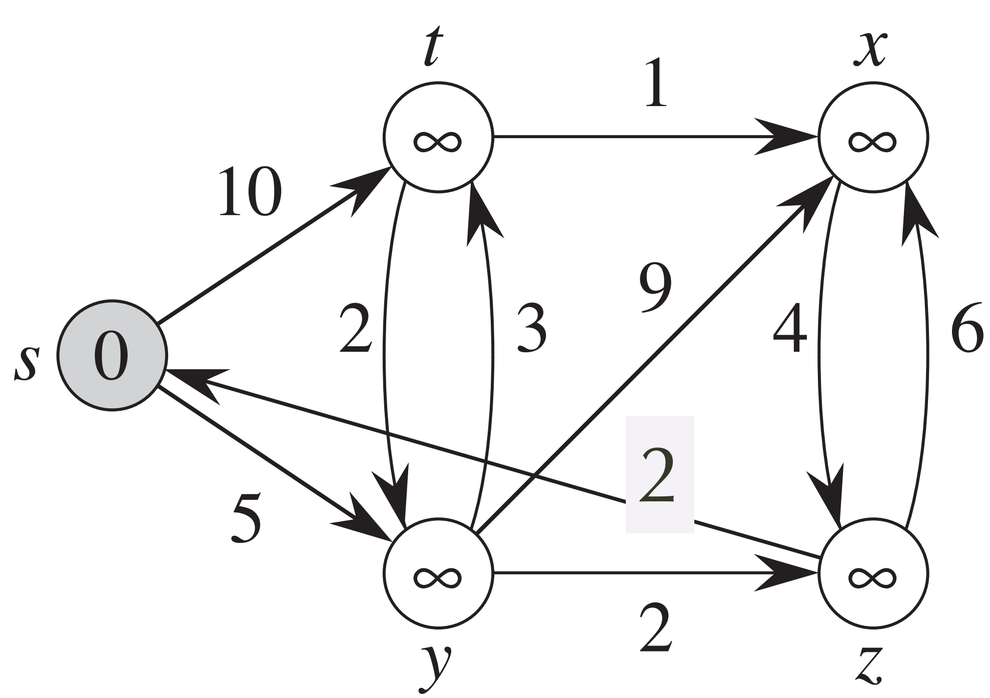
Floyd-Warshall algorithm



$$
\begin{array}{c}
\\
\begin{matrix} s \\ t \\ x \\ y \\ z \end{matrix}
\end{array}
\begin{array}{c}
\begin{matrix} s & t & x & y & z \end{matrix} \\
\begin{bmatrix}
0 & 10 & \infty & 5 & \infty \\
\infty & 0 & 1 & 2 & \infty \\
\infty & \infty & 0 & \infty & 4 \\
\infty & 3 & 9 & 0 & 2 \\
2 & \infty & 6 & \infty & 0
\end{bmatrix}
\end{array}
$$

The shortest path from u to v that passes none vertex

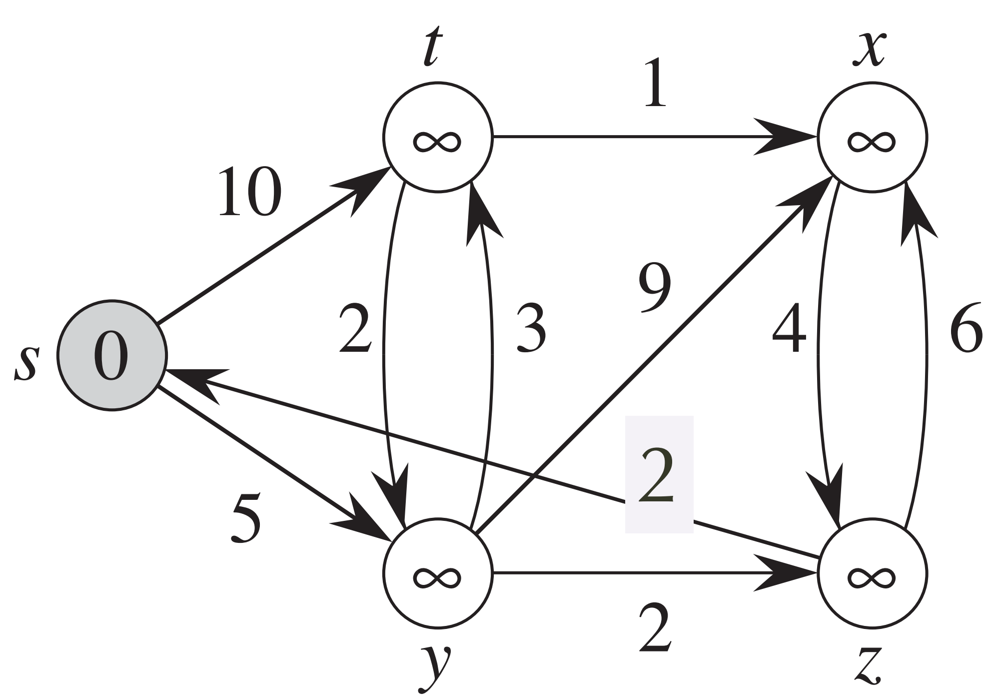
Floyd-Warshall algorithm



$$
\begin{array}{c|c}
 & \begin{array}{ccccc} s & t & x & y & z \end{array} \\
\begin{array}{c} s \\ t \\ x \\ y \\ z \end{array} &
\begin{bmatrix}
0 & 10 & \infty & 5 & \infty \\
\infty & 0 & 1 & 2 & \infty \\
\infty & \infty & 0 & \infty & 4 \\
\infty & 3 & 9 & 0 & 2 \\
2 & \infty & 6 & \infty & 0
\end{bmatrix}
\end{array}
$$

The shortest path from u to v that passes none vertex

Floyd-Warshall algorithm

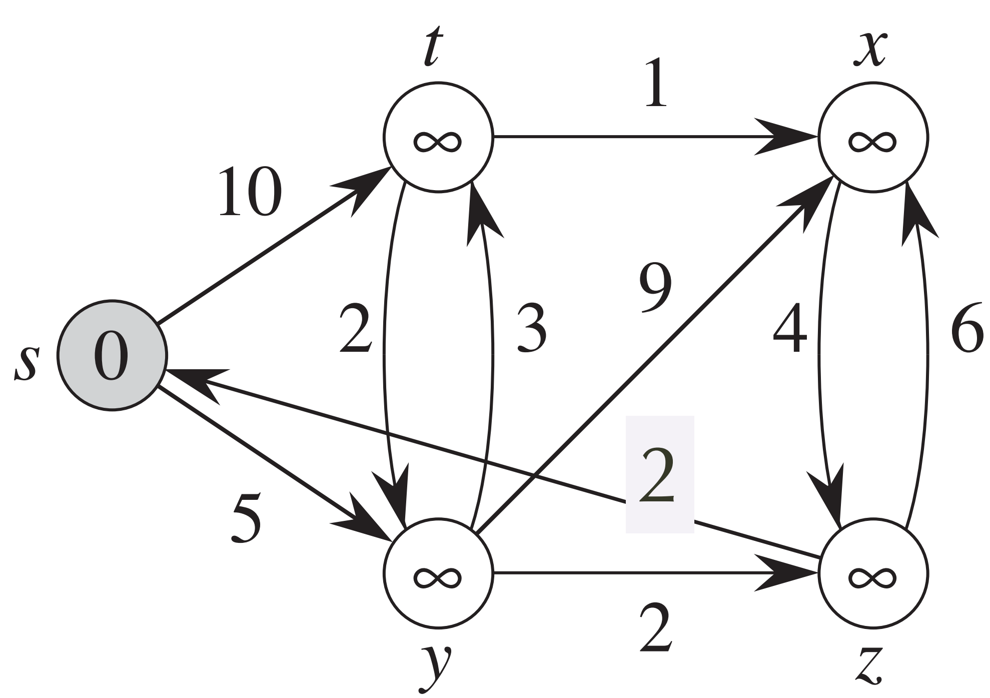


$$
\begin{array}{c}
\\
s\\ t\\ x\\ y\\ z
\end{array}
\begin{array}{c}
\begin{array}{ccccc} s & t & x & y & z \end{array}\\
\begin{bmatrix}
0 & 10 & \infty & 5 & \infty\\
\infty & 0 & 1 & 2 & \infty\\
\infty & \infty & 0 & \infty & 4\\
\infty & 3 & 9 & 0 & 2\\
2 & \boxed{\mathbf{12}} & 6 & \infty & 0
\end{bmatrix}
\end{array}
$$

The shortest path from u to v that passes none vertex

Xiaojuan Cai, School of Software, SJTU.

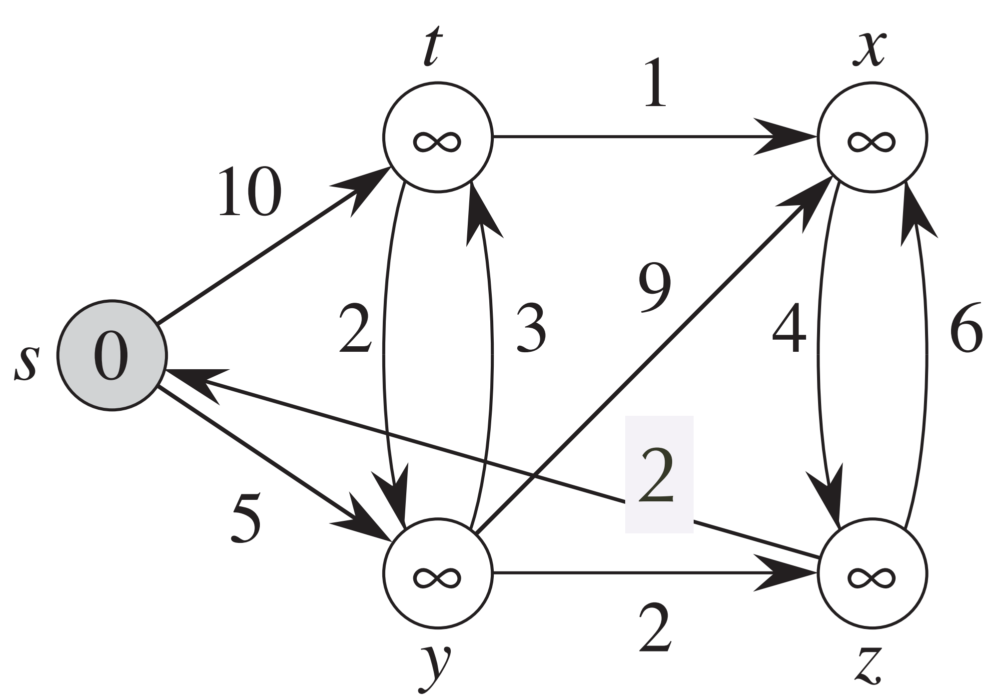
Floyd-Warshall algorithm



$$
\begin{array}{c|c}
 & \begin{array}{ccccc} s & t & x & y & z \end{array} \\
\begin{array}{c} s \\ t \\ x \\ y \\ z \end{array} &
\begin{bmatrix}
0 & 10 & \infty & 5 & \infty \\
\infty & 0 & 1 & 2 & \infty \\
\infty & \infty & 0 & \infty & 4 \\
\infty & 3 & 9 & 0 & 2 \\
2 & 12 & 6 & \infty & 0
\end{bmatrix}
\end{array}
$$

The shortest path from u to v that passes none vertex

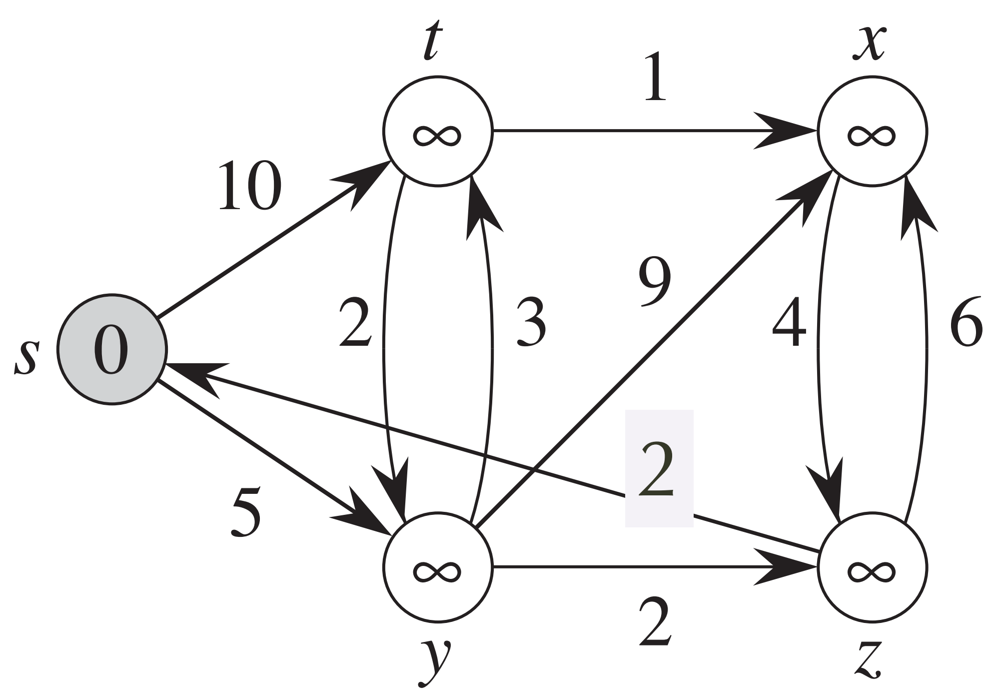
Floyd-Warshall algorithm



$$
\begin{array}{c}
 \\ s \\ t \\ x \\ y \\ z
\end{array}
\begin{array}{c}
\begin{array}{ccccc} s & t & x & y & z \end{array} \\
\begin{bmatrix}
0 & 10 & \infty & 5 & \infty \\
\infty & 0 & 1 & 2 & \infty \\
\infty & \infty & 0 & \infty & 4 \\
\infty & 3 & 9 & 0 & 2 \\
2 & \boxed{12} & 6 & \boxed{7} & 0
\end{bmatrix}
\end{array}
$$

The shortest path from u to v that passes none vertex

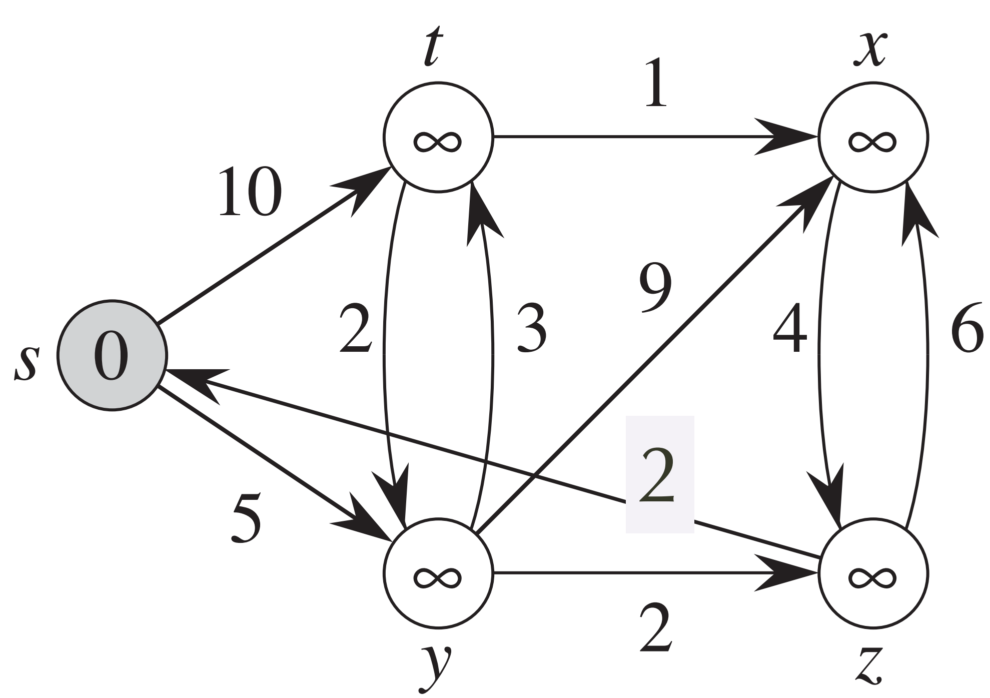
Floyd-Warshall algorithm



$$\begin{array}{c} \\ s \\ t \\ x \\ y \\ z \end{array}\begin{array}{c} \begin{array}{ccccc} s & t & x & y & z \end{array} \\ \begin{bmatrix} 0 & 10 & \infty & 5 & \infty \\ \infty & 0 & 1 & 2 & \infty \\ \infty & \infty & 0 & \infty & 4 \\ \infty & 3 & 9 & 0 & 2 \\ 2 & 12 & 6 & 7 & 0 \end{bmatrix} \end{array}$$

The shortest path from u to v that passes none vertex

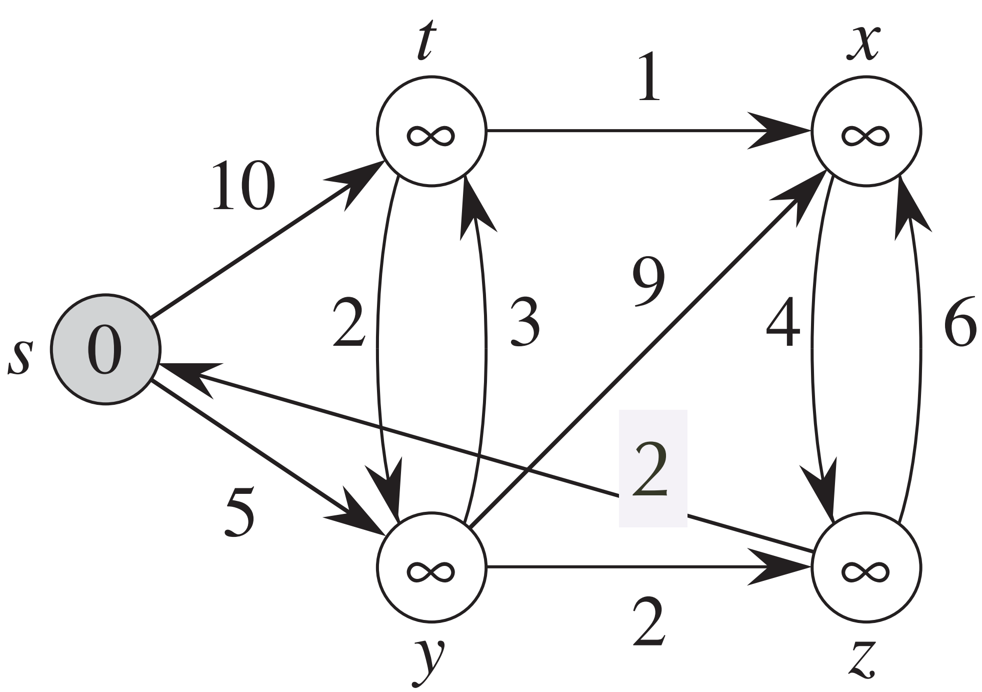
Floyd-Warshall algorithm



$$\begin{array}{c} \\ s \\ t \\ x \\ y \\ z \end{array}\begin{array}{c} \begin{array}{ccccc} s & t & x & y & z \end{array} \\ \begin{bmatrix} 0 & 10 & \infty & 5 & \infty \\ \infty & 0 & 1 & 2 & \infty \\ \infty & \infty & 0 & \infty & 4 \\ \infty & 3 & 9 & 0 & 2 \\ 2 & \mathbf{12} & 6 & \mathbf{7} & 0 \end{bmatrix} \end{array}$$

The shortest path from u to v that may passes ***s***

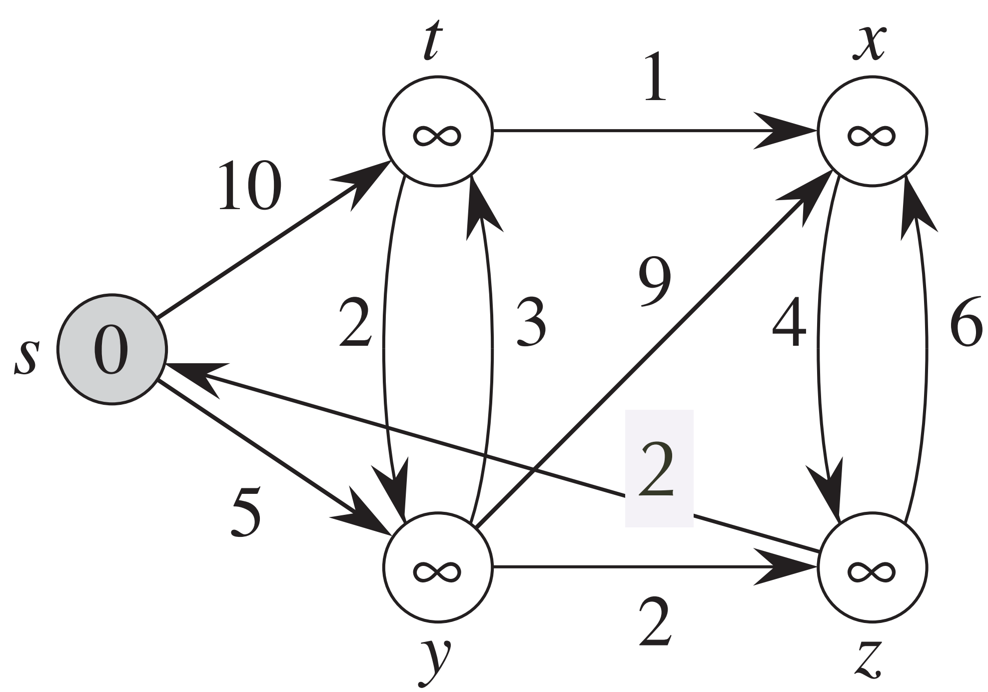
Floyd-Warshall algorithm



$$
\begin{array}{c} \\ s \\ t \\ x \\ y \\ z \end{array}
\begin{array}{c}
\begin{array}{ccccc} s & t & x & y & z \end{array} \\
\begin{bmatrix}
0 & 10 & \infty & 5 & \infty \\
\infty & 0 & 1 & 2 & \infty \\
\infty & \infty & 0 & \infty & 4 \\
\infty & 3 & 9 & 0 & 2 \\
2 & 12 & 6 & 7 & 0
\end{bmatrix}
\end{array}
$$

The shortest path from u to v that may passes ***s***

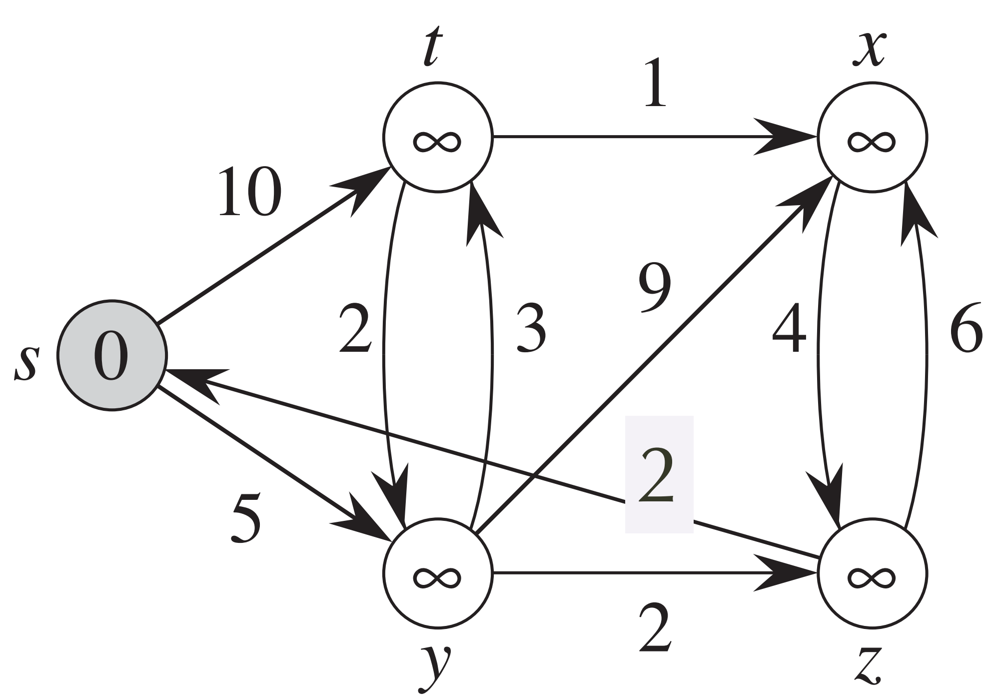
Floyd-Warshall algorithm



$$
\begin{array}{c|ccccc}
 & s & t & x & y & z \\
\hline
s & 0 & 10 & \boxed{11} & 5 & \infty \\
t & \infty & 0 & 1 & 2 & \infty \\
x & \infty & \infty & 0 & \infty & 4 \\
y & \infty & 3 & 9 & 0 & 2 \\
z & 2 & \boxed{12} & 6 & \boxed{7} & 0
\end{array}
$$

The shortest path from u to v that may passes ***s***

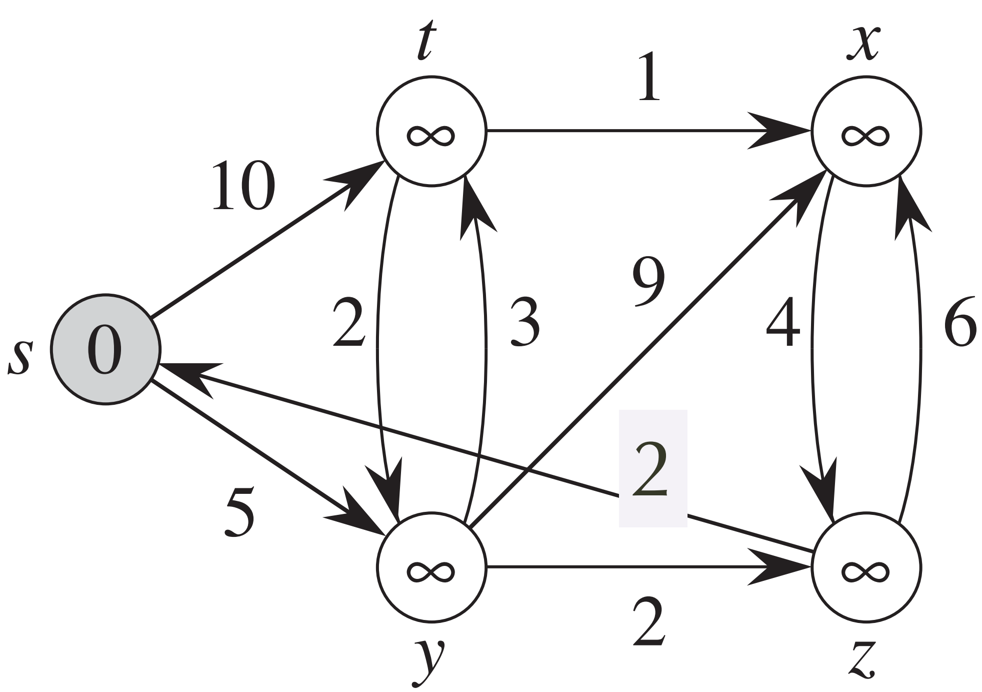
Floyd-Warshall algorithm



$$
\begin{array}{c c}
 & \begin{array}{ccccc} s & t & x & y & z \end{array} \\
\begin{array}{c} s \\ t \\ x \\ y \\ z \end{array} &
\begin{bmatrix}
0 & 10 & 11 & 5 & \infty \\
\infty & 0 & 1 & 2 & \infty \\
\infty & \infty & 0 & \infty & 4 \\
\infty & 3 & 4 & 0 & 2 \\
2 & 12 & 6 & 7 & 0
\end{bmatrix}
\end{array}
$$

The shortest path from u to v that may passes ***s***

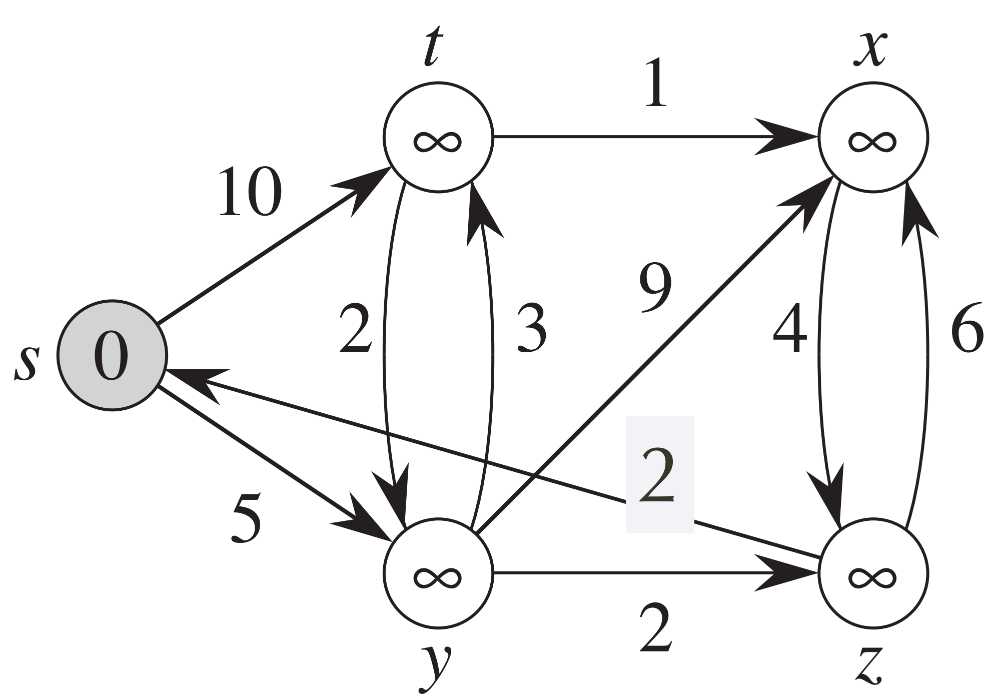
Floyd-Warshall algorithm



$$
\begin{array}{c}
\\ s \\ t \\ x \\ y \\ z
\end{array}
\begin{array}{c}
\begin{array}{ccccc} s & t & x & y & z \end{array} \\
\begin{bmatrix}
0 & 10 & \mathbf{11} & 5 & \infty \\
\infty & 0 & 1 & 2 & \infty \\
\infty & \infty & 0 & \infty & 4 \\
\infty & 3 & \mathbf{4} & 0 & 2 \\
2 & \mathbf{12} & 6 & \mathbf{7} & 0
\end{bmatrix}
\end{array}
$$

The shortest path from u to v that may passes ***s***

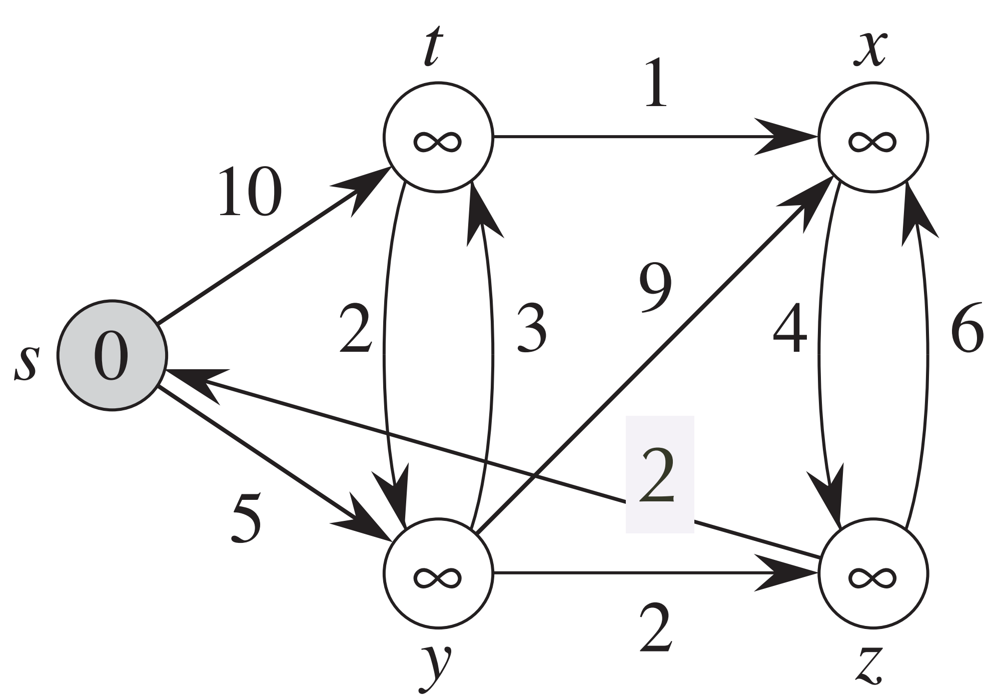
Floyd-Warshall algorithm



$$
\begin{array}{c|c}
 & \begin{array}{ccccc} s & t & x & y & z \end{array} \\
\begin{array}{c} s \\ t \\ x \\ y \\ z \end{array} &
\begin{bmatrix}
0 & 10 & \boxed{\mathbf{11}} & 5 & \infty \\
\infty & 0 & 1 & 2 & \infty \\
\infty & \infty & 0 & \infty & 4 \\
\infty & 3 & \boxed{\mathbf{4}} & 0 & 2 \\
2 & \boxed{\mathbf{12}} & 6 & \boxed{\mathbf{7}} & 0
\end{bmatrix}
\end{array}
$$

The shortest path from u to v that may passes ***s, t***

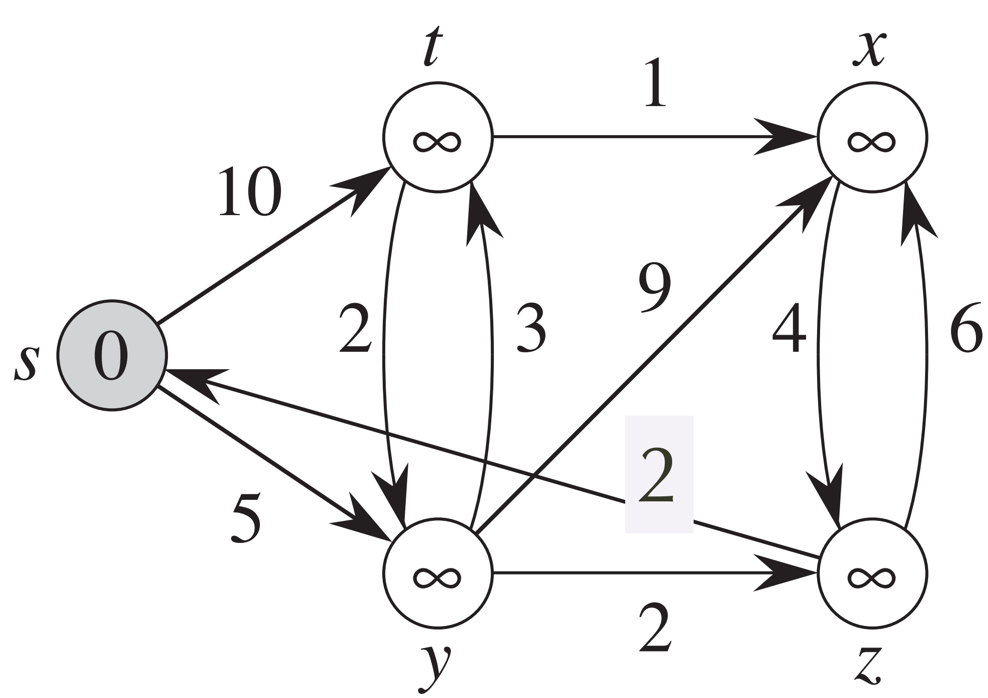
Floyd-Warshall algorithm



$$
\begin{array}{c|ccccc}
 & s & t & x & y & z \\
\hline
s & 0 & 10 & \mathbf{11} & 5 & \infty \\
t & \infty & 0 & 1 & 2 & \infty \\
x & \infty & \infty & 0 & \infty & 4 \\
y & \infty & 3 & \mathbf{4} & 0 & 2 \\
z & 2 & \mathbf{12} & 6 & \mathbf{7} & 0
\end{array}
$$

The shortest path from u to v that may passes ***s, t***

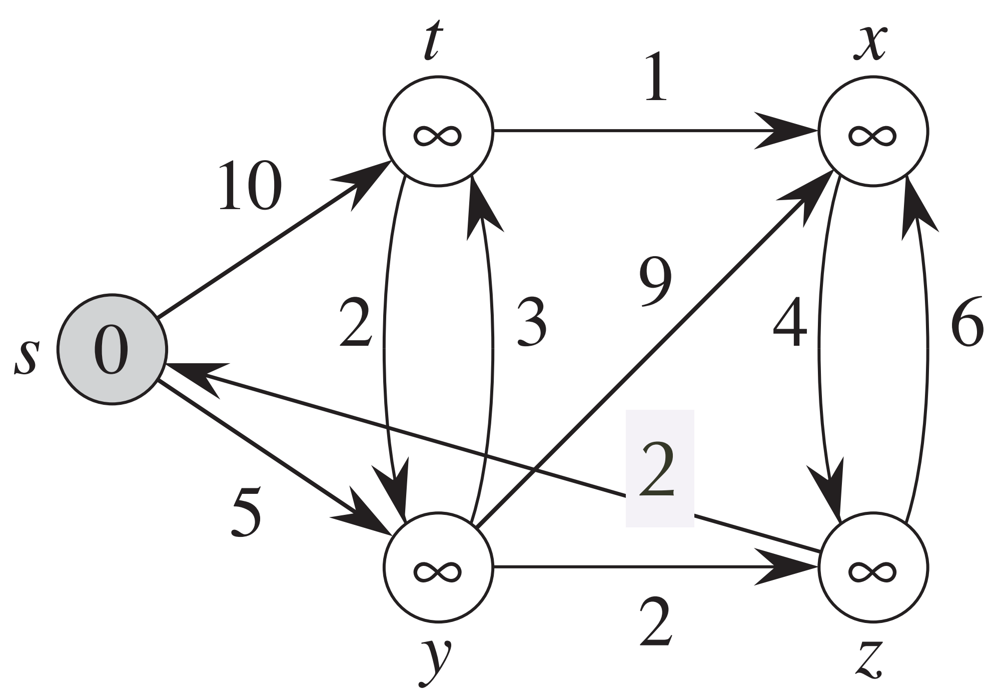
Floyd-Warshall algorithm



$$
\begin{array}{c|ccccc}
 & s & t & x & y & z \\
\hline
s & 0 & 10 & \mathbf{11} & 5 & \mathbf{15} \\
t & \infty & 0 & 1 & 2 & \infty \\
x & \infty & \infty & 0 & \infty & 4 \\
y & \infty & 3 & \mathbf{4} & 0 & 2 \\
z & 2 & \mathbf{12} & 6 & \mathbf{7} & 0
\end{array}
$$

The shortest path from u to v that may passes ***s, t***

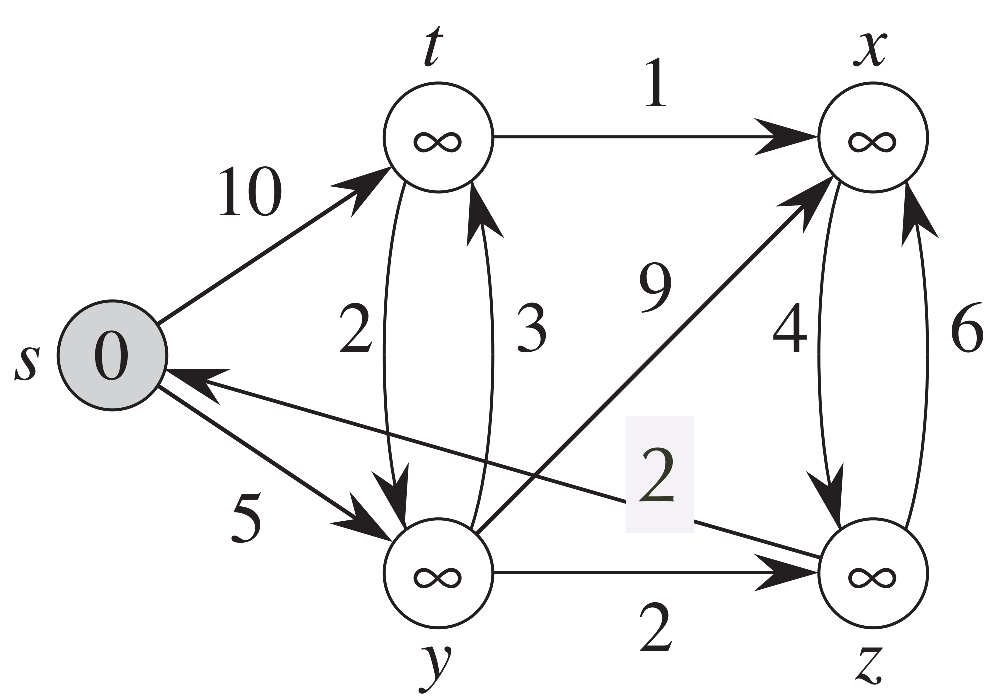
Floyd-Warshall algorithm



$$
\begin{array}{c}
 \\ s \\ t \\ x \\ y \\ z
\end{array}
\begin{array}{c}
\begin{array}{ccccc} s & t & x & y & z \end{array} \\
\begin{bmatrix}
0 & 10 & \mathbf{11} & 5 & \mathbf{15} \\
\infty & 0 & 1 & 2 & \mathbf{5} \\
\infty & \infty & 0 & \infty & 4 \\
\infty & 3 & \mathbf{4} & 0 & 2 \\
2 & \mathbf{12} & 6 & \mathbf{7} & 0
\end{bmatrix}
\end{array}
$$

The shortest path from u to v that may passes ***s, t***

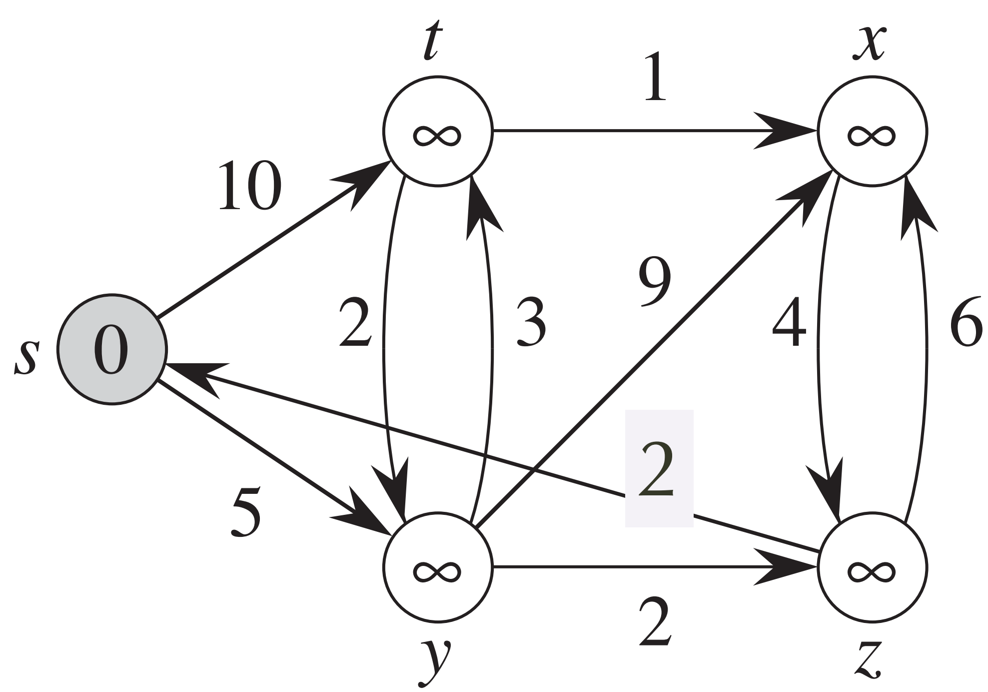
Floyd-Warshall algorithm



$$\begin{array}{c|c} & \begin{matrix} s & t & x & y & z \end{matrix} \\ \begin{matrix} s \\ t \\ x \\ y \\ z \end{matrix} & \begin{bmatrix} 0 & 10 & \mathbf{11} & 5 & \mathbf{15} \\ \infty & 0 & 1 & 2 & \mathbf{5} \\ \infty & \infty & 0 & \infty & 4 \\ \infty & 3 & \mathbf{4} & 0 & 2 \\ 2 & \mathbf{12} & 6 & \mathbf{7} & 0 \end{bmatrix} \end{array}$$

The shortest path from u to v that may passes ***s, t***

Floyd-Warshall algorithm

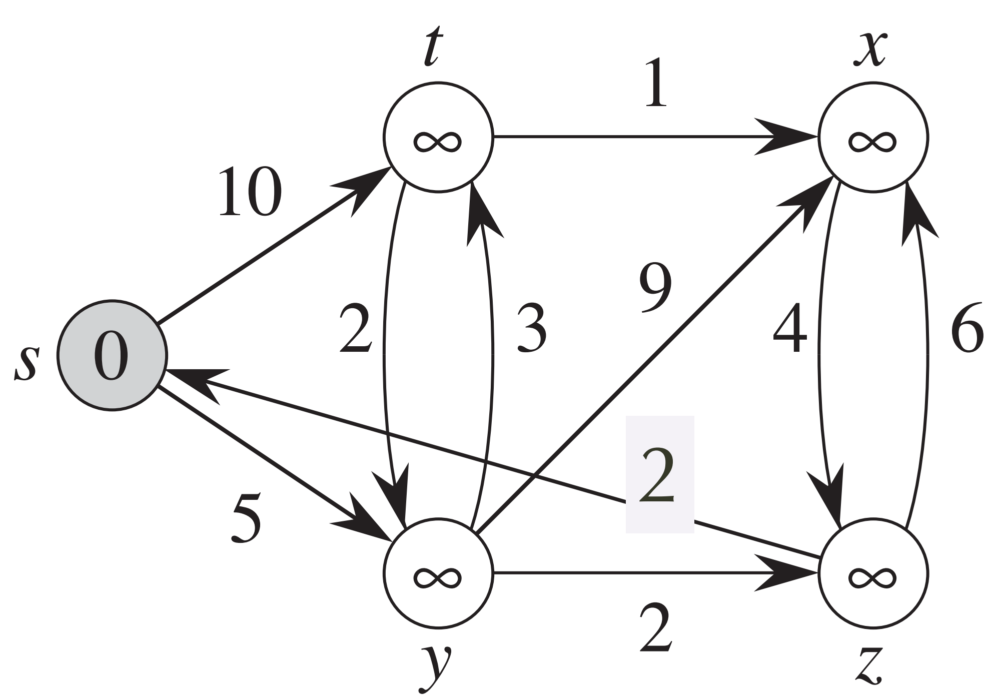


$$
\begin{array}{c|ccccc}
 & \color{blue}{s} & \color{blue}{t} & \color{blue}{x} & \color{blue}{y} & \color{blue}{z} \\
\hline
\end{array}
$$

$$
\begin{array}{c}
\color{purple}{s} \\ \color{purple}{t} \\ \color{purple}{x} \\ \color{purple}{y} \\ \color{purple}{z}
\end{array}
\begin{bmatrix}
0 & 10 & \mathbf{11} & 5 & \mathbf{15} \\
\infty & 0 & 1 & 2 & \mathbf{5} \\
\infty & \infty & 0 & \infty & 4 \\
\infty & 3 & \mathbf{4} & 0 & 2 \\
2 & \mathbf{12} & 6 & \mathbf{7} & 0
\end{bmatrix}
$$

The shortest path from u to v that may passes ***s, t, x***

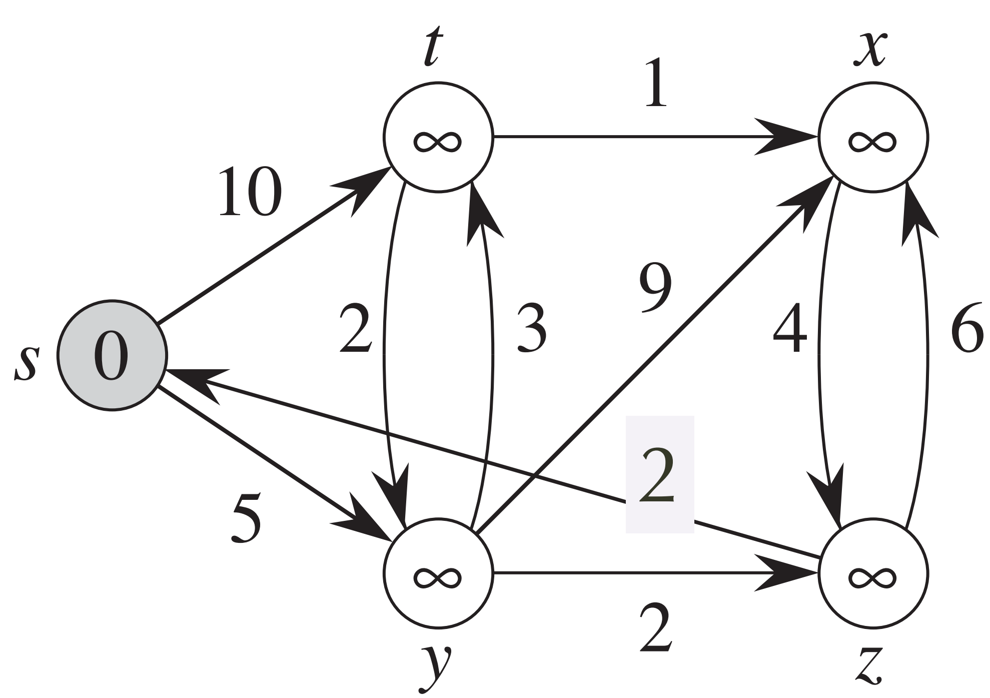
Floyd-Warshall algorithm



$$
\begin{array}{c|ccccc}
 & s & t & x & y & z \\
\hline
s & 0 & 10 & 11 & 5 & 15 \\
t & \infty & 0 & 1 & 2 & 5 \\
x & \infty & \infty & 0 & \infty & 4 \\
y & \infty & 3 & 4 & 0 & 2 \\
z & 2 & 12 & 6 & 7 & 0
\end{array}
$$

The shortest path from u to v that may passes ***s, t, x***

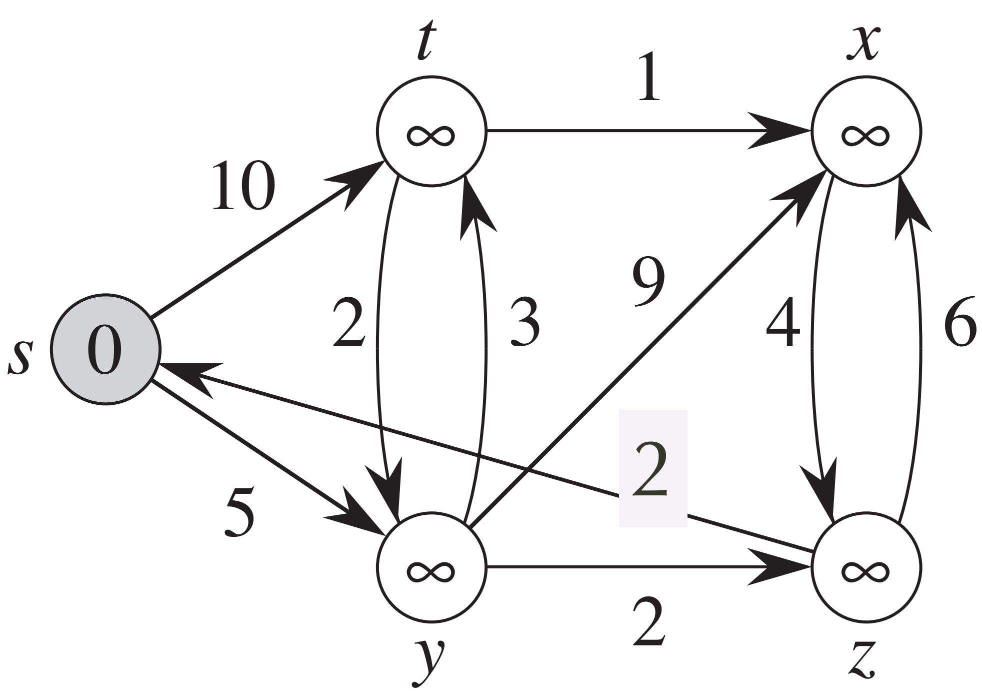
Floyd-Warshall algorithm



$$
\begin{array}{c} \\ s \\ t \\ x \\ y \\ z \end{array}
\begin{array}{c}
\begin{array}{ccccc} s & t & x & y & z \end{array} \\
\begin{bmatrix}
0 & 10 & 11 & 5 & 7 \\
\infty & 0 & 1 & 2 & 5 \\
\infty & \infty & 0 & \infty & 4 \\
\infty & 3 & 4 & 0 & 2 \\
2 & 12 & 6 & 7 & 0
\end{bmatrix}
\end{array}
$$

The shortest path from u to v that may passes ***s, t, x***

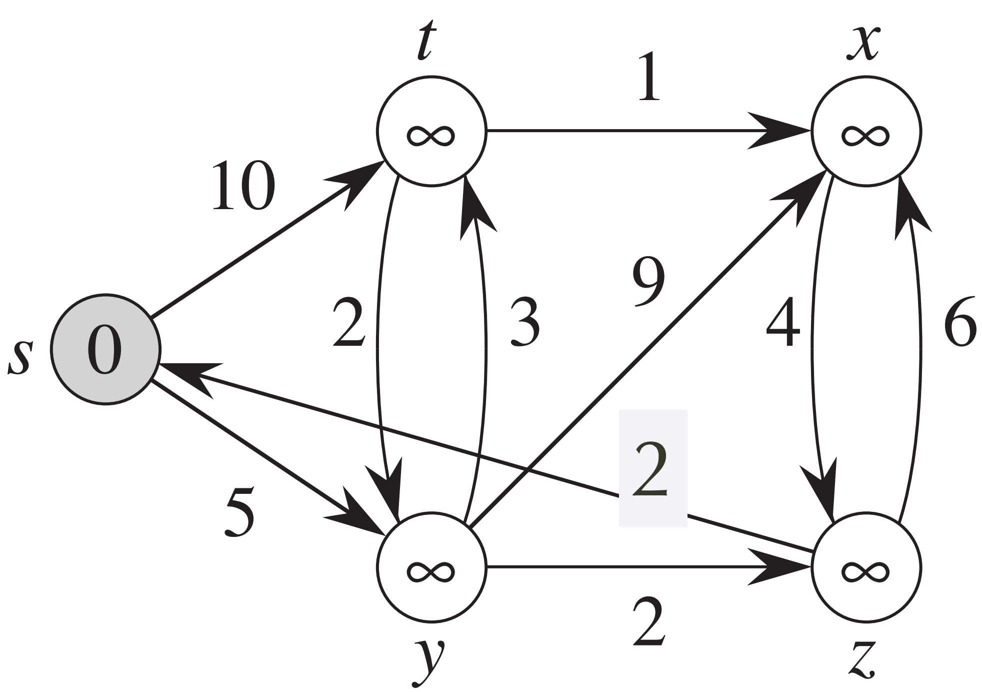
Floyd-Warshall algorithm



$$
\begin{array}{c|c}
 & \begin{array}{ccccc} s & t & x & y & z \end{array} \\
\begin{array}{c} s \\ t \\ x \\ y \\ z \end{array} &
\begin{bmatrix}
0 & 10 & \mathbf{11} & 5 & \mathbf{7} \\
\infty & 0 & 1 & 2 & \mathbf{4} \\
\infty & \infty & 0 & \infty & 4 \\
\infty & 3 & \mathbf{4} & 0 & 2 \\
2 & \mathbf{12} & 6 & \mathbf{7} & 0
\end{bmatrix}
\end{array}
$$

The shortest path from u to v that may passes ***s, t, x***

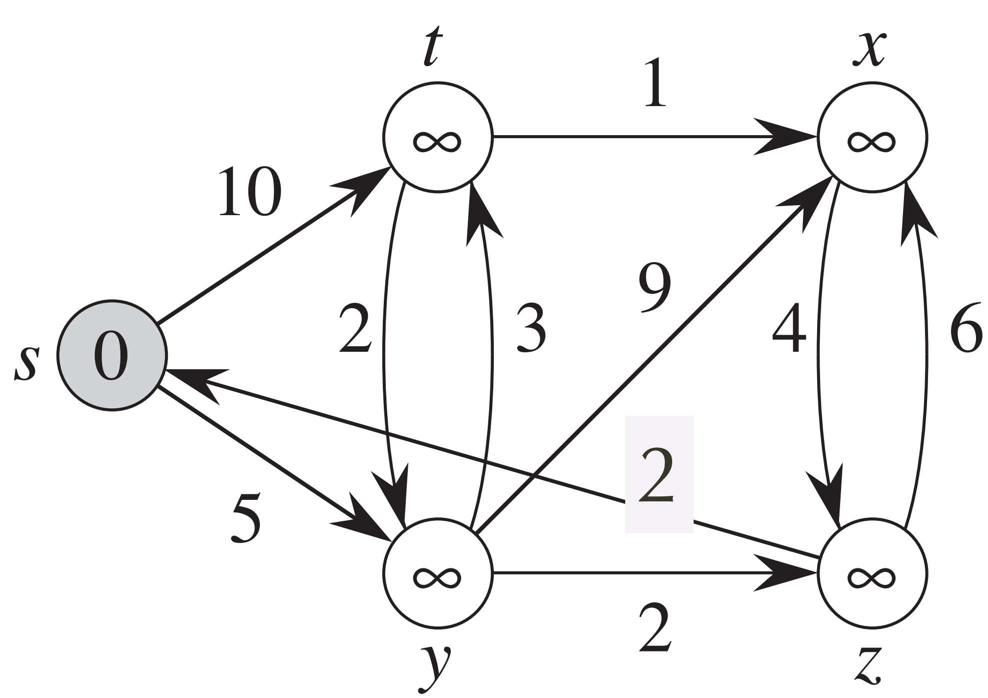
Floyd-Warshall algorithm



$$
\begin{array}{c|c}
 & \begin{matrix} s & t & x & y & z \end{matrix} \\
\begin{matrix} s \\ t \\ x \\ y \\ z \end{matrix} &
\begin{bmatrix}
0 & 10 & \mathbf{11} & 5 & \mathbf{7} \\
\infty & 0 & 1 & 2 & \mathbf{4} \\
\infty & \infty & 0 & \infty & 4 \\
\infty & 3 & \mathbf{4} & 0 & 2 \\
2 & \mathbf{10} & 6 & \mathbf{7} & 0
\end{bmatrix}
\end{array}
$$

The shortest path from u to v that may passes ***s, t, x***

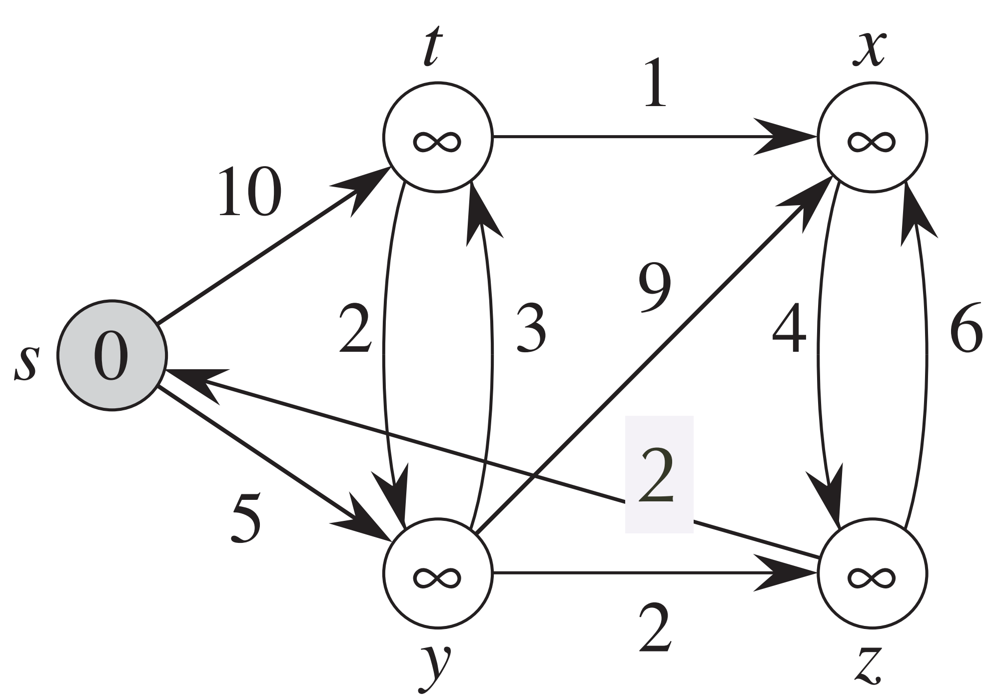
Floyd-Warshall algorithm



$$
\begin{array}{c|c}
 & \begin{array}{ccccc} s & t & x & y & z \end{array} \\
\begin{array}{c} s \\ t \\ x \\ y \\ z \end{array} & \begin{bmatrix} 0 & 8 & 11 & 5 & 7 \\ \infty & 0 & 1 & 2 & 4 \\ \infty & \infty & 0 & \infty & 4 \\ \infty & 3 & 4 & 0 & 2 \\ 2 & 10 & 6 & 7 & 0 \end{bmatrix}
\end{array}
$$

The shortest path from u to v that may passes ***s, t, x***

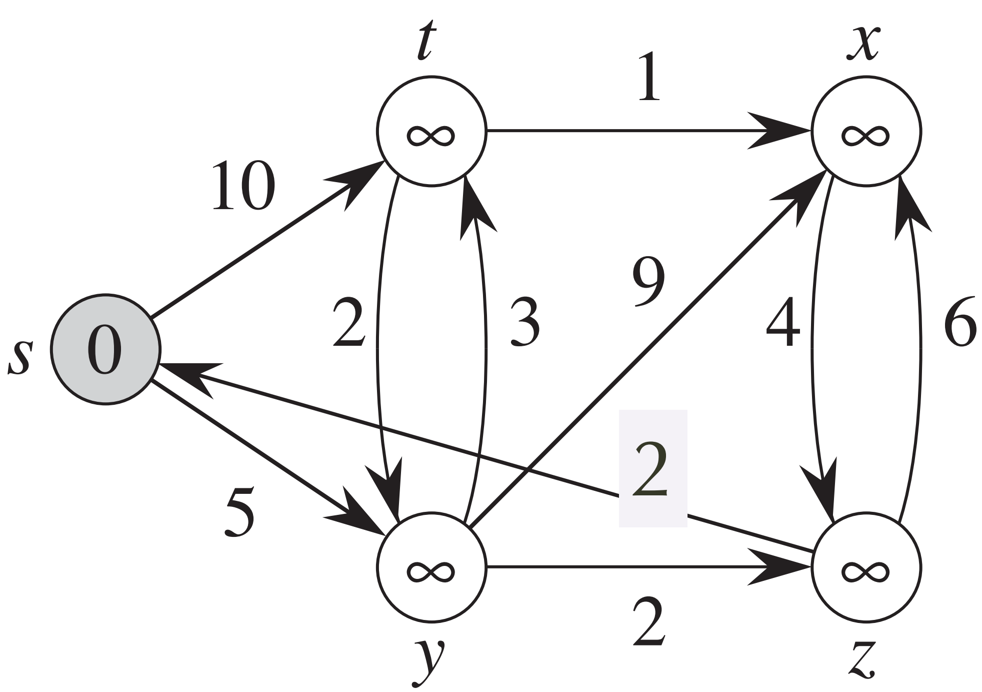
Floyd-Warshall algorithm



$$
\begin{array}{c|c}
 & \begin{array}{ccccc} s & t & x & y & z \end{array} \\
\begin{array}{c} s \\ t \\ x \\ y \\ z \end{array} &
\begin{bmatrix}
0 & \mathbf{8} & \mathbf{9} & 5 & \mathbf{7} \\
\infty & 0 & 1 & 2 & \mathbf{4} \\
\infty & \infty & 0 & \infty & 4 \\
\infty & 3 & \mathbf{4} & 0 & 2 \\
2 & \mathbf{10} & 6 & \mathbf{7} & 0
\end{bmatrix}
\end{array}
$$

The shortest path from u to v that may passes ***s, t, x***

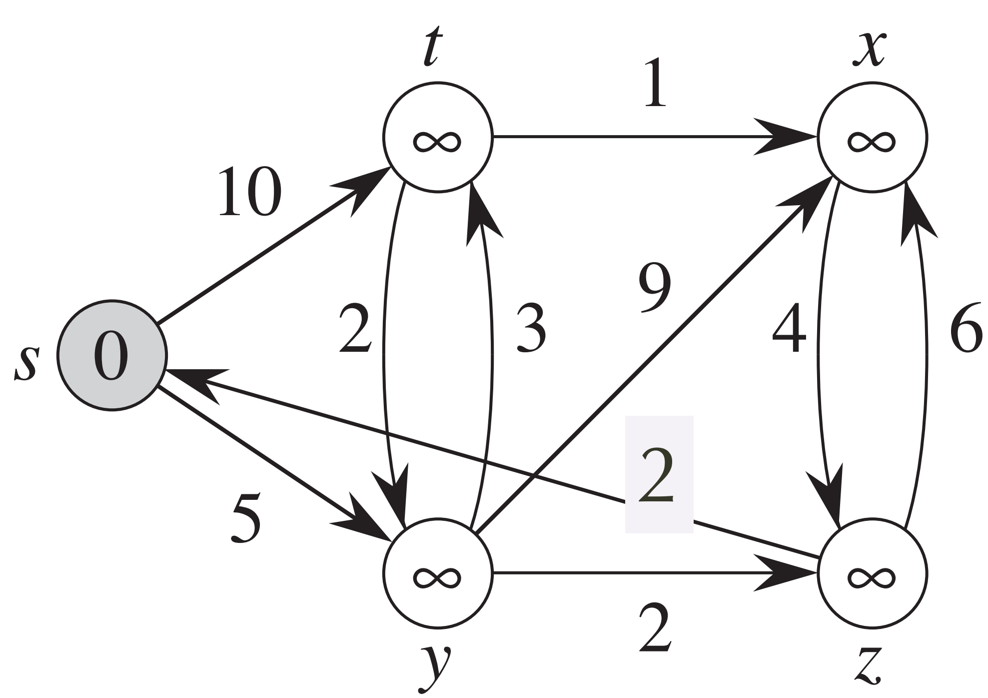
Floyd-Warshall algorithm



$$
\begin{array}{c|c}
 & \begin{array}{ccccc} s & t & x & y & z \end{array} \\
\begin{array}{c} s \\ t \\ x \\ y \\ z \end{array} &
\begin{bmatrix}
0 & \mathbf{8} & \mathbf{9} & 5 & \mathbf{7} \\
\infty & 0 & 1 & 2 & \mathbf{4} \\
\infty & \infty & 0 & \infty & 4 \\
\infty & 3 & \mathbf{4} & 0 & 2 \\
2 & \mathbf{10} & 6 & \mathbf{7} & 0
\end{bmatrix}
\end{array}
$$

The shortest path from u to v that may passes ***s, t, x***

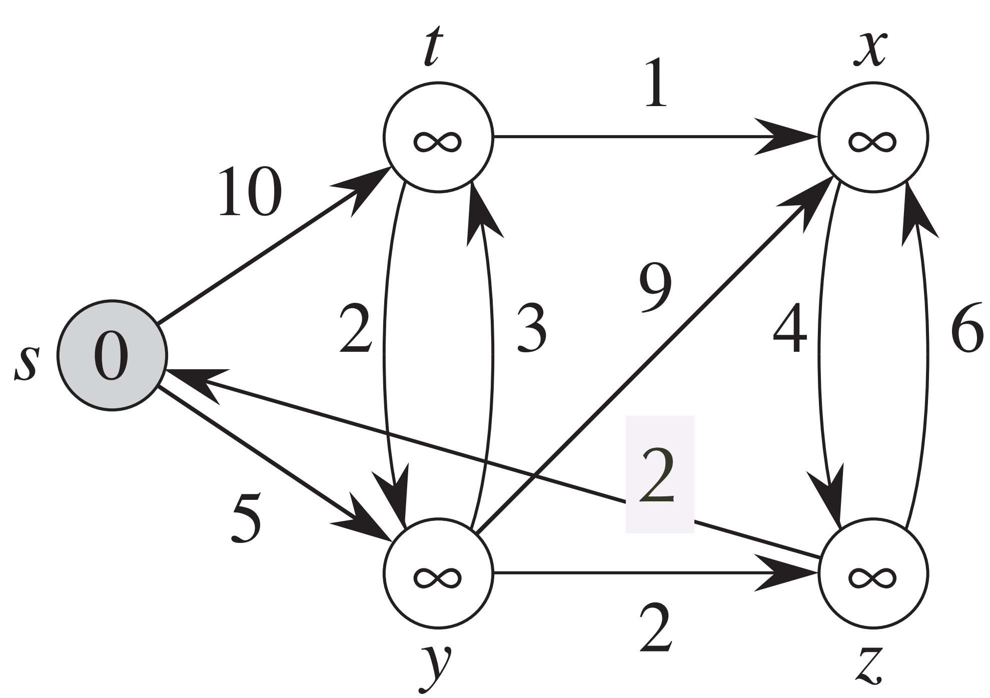
Floyd-Warshall algorithm



$$
\begin{array}{c} \\ s \\ t \\ x \\ y \\ z \end{array}
\begin{array}{c}
\begin{array}{ccccc} s & t & x & y & z \end{array} \\
\begin{bmatrix}
0 & 8 & 9 & 5 & 7 \\
\infty & 0 & 1 & 2 & 4 \\
\infty & \infty & 0 & \infty & 4 \\
\infty & 3 & 4 & 0 & 2 \\
2 & 10 & 6 & 7 & 0
\end{bmatrix}
\end{array}
$$

The shortest path from u to v that may passes ***s, t, x, y***

Floyd-Warshall algorithm

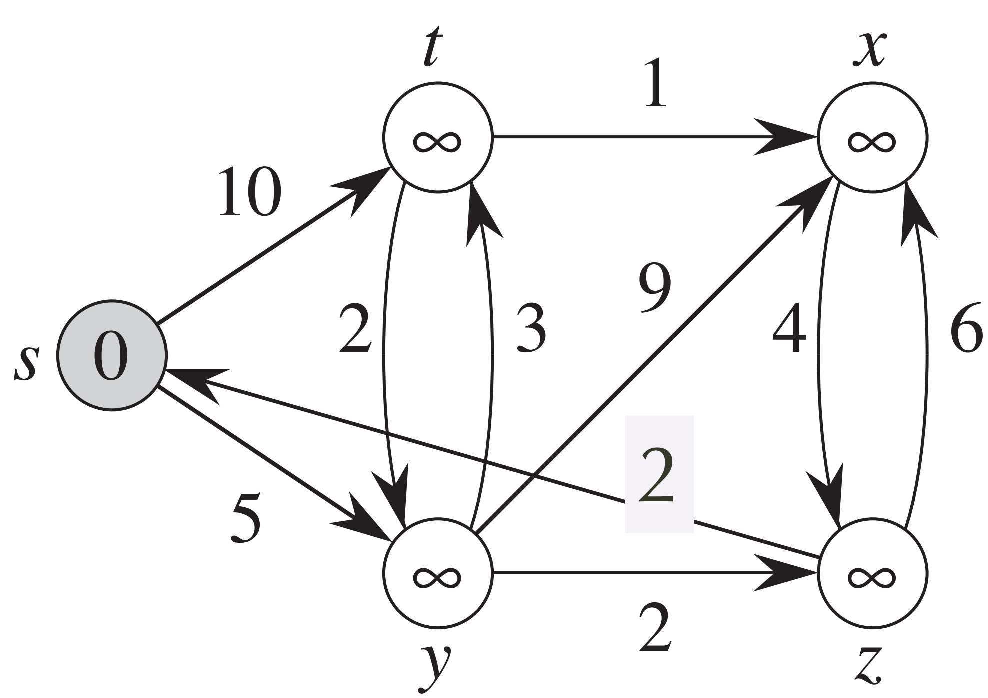


$$
\begin{array}{c} \\ s \\ t \\ x \\ y \\ z \end{array}
\begin{array}{c}
\begin{array}{ccccc} s & t & x & y & z \end{array} \\
\begin{bmatrix}
0 & 8 & 9 & 5 & 7 \\
\infty & 0 & 1 & 2 & 4 \\
\infty & \infty & 0 & \infty & 4 \\
\infty & 3 & 4 & 0 & 2 \\
2 & 10 & 6 & 7 & 0
\end{bmatrix}
\end{array}
$$

The shortest path from u to v that may passes ***s, t, x, y***

Xiaojuan Cai, School of Software, SJTU.

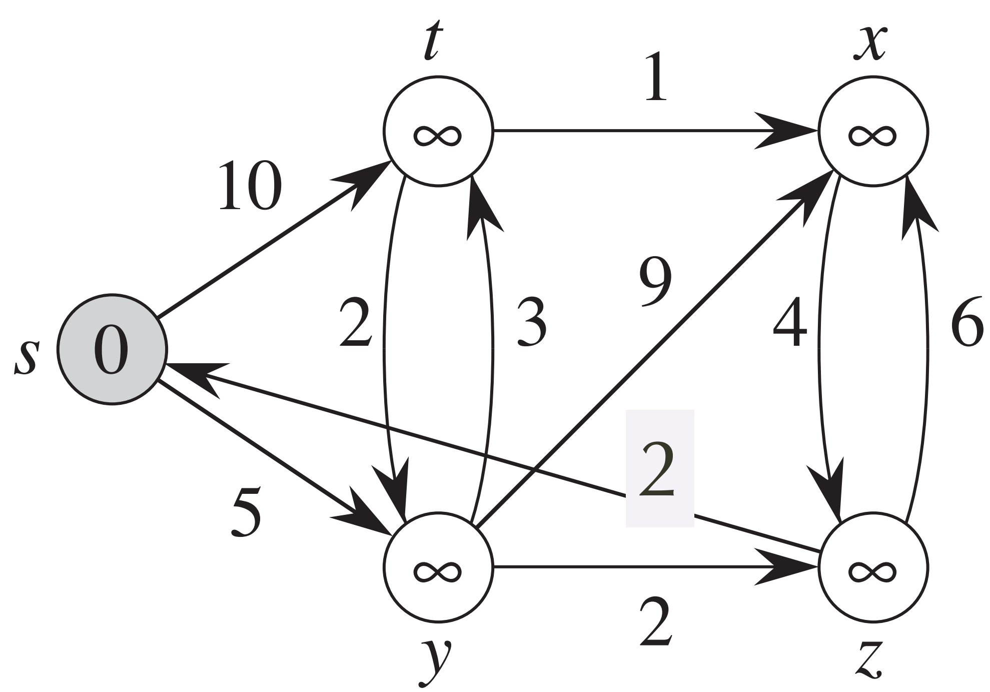
Floyd-Warshall algorithm



$$
\begin{array}{c|c}
 & \begin{array}{ccccc} s & t & x & y & z \end{array} \\
\begin{array}{c} s \\ t \\ x \\ y \\ z \end{array} &
\left[\begin{array}{ccccc}
0 & \mathbf{8} & \mathbf{9} & 5 & 7 \\
\infty & 0 & 1 & 2 & 4 \\
\infty & \infty & 0 & \mathbf{11} & 4 \\
\infty & 3 & \mathbf{4} & 0 & 2 \\
2 & \mathbf{10} & 6 & 7 & 0
\end{array}\right]
\end{array}
$$

The shortest path from u to v that may passes ***s, t, x, y***

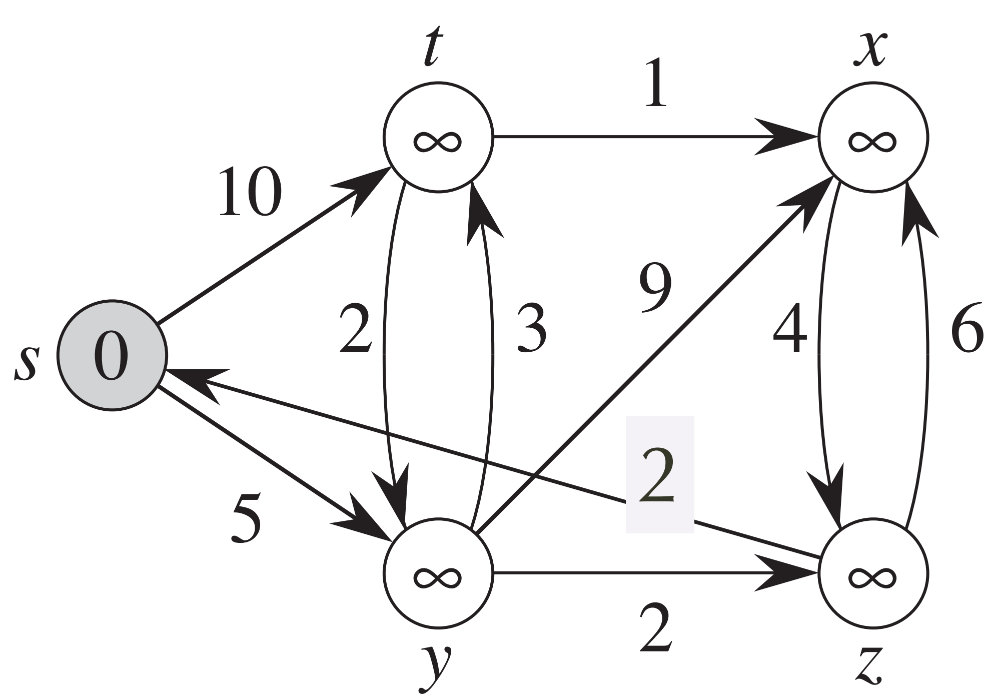
Floyd-Warshall algorithm



$$
\begin{array}{c|ccccc}
 & s & t & x & y & z \\
\hline
s & 0 & \mathbf{8} & \mathbf{9} & 5 & 7 \\
t & \infty & 0 & 1 & 2 & \mathbf{4} \\
x & \infty & \infty & 0 & \mathbf{11} & 4 \\
y & \mathbf{4} & 3 & \mathbf{4} & 0 & 2 \\
z & 2 & \mathbf{10} & 6 & 7 & 0
\end{array}
$$

The shortest path from u to v that may passes ***s, t, x, y***

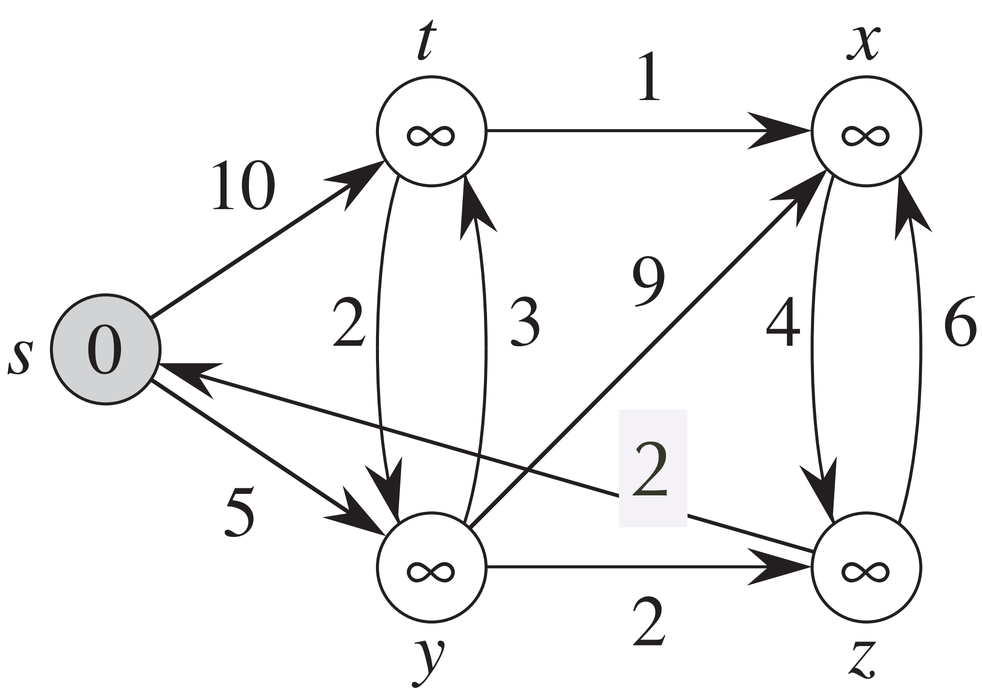
Floyd-Warshall algorithm



$$
\begin{array}{c|c}
 & \begin{array}{ccccc} s & t & x & y & z \end{array} \\
\begin{array}{c} s \\ t \\ x \\ y \\ z \end{array} &
\begin{bmatrix}
0 & 8 & 9 & 5 & 7 \\
\infty & 0 & 1 & 2 & 4 \\
\infty & 14 & 0 & 11 & 4 \\
4 & 3 & 4 & 0 & 2 \\
2 & 10 & 6 & 7 & 0
\end{bmatrix}
\end{array}
$$

The shortest path from u to v that may passes ***s, t, x, y***

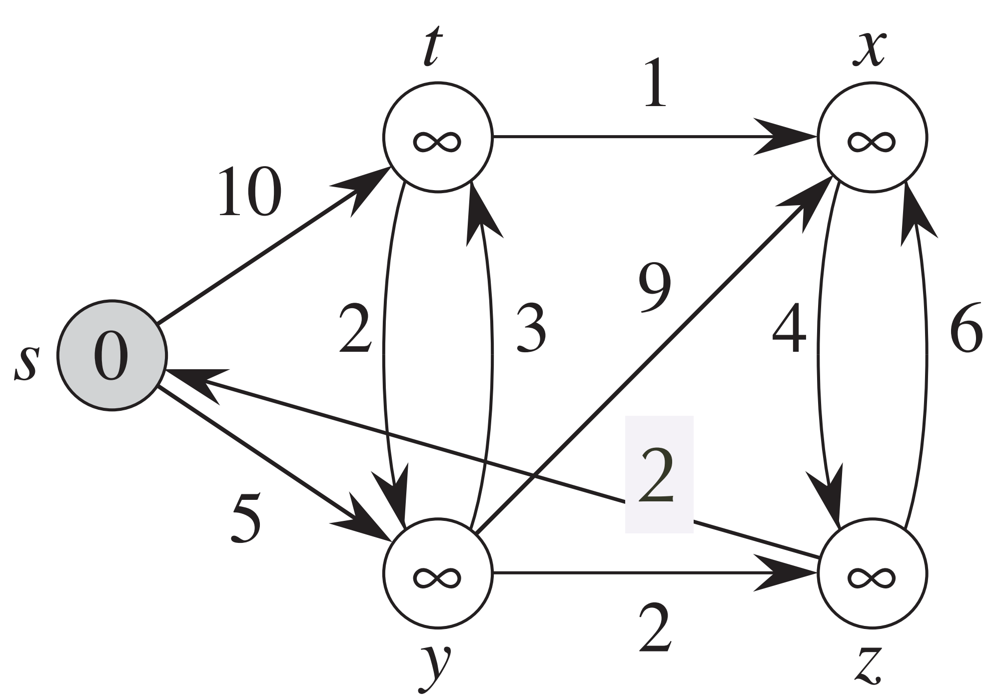
Floyd-Warshall algorithm



$$
\begin{array}{c} \\ s \\ t \\ x \\ y \\ z \end{array}
\begin{array}{c}
\begin{array}{ccccc} s & t & x & y & z \end{array} \\
\begin{bmatrix}
0 & \mathbf{8} & \mathbf{9} & 5 & 7 \\
\infty & 0 & 1 & 2 & 4 \\
\mathbf{6} & \mathbf{14} & 0 & \mathbf{11} & 4 \\
\mathbf{4} & 3 & \mathbf{4} & 0 & 2 \\
2 & 10 & 6 & 7 & 0
\end{bmatrix}
\end{array}
$$

The shortest path from u to v that may passes ***s, t, x, y***

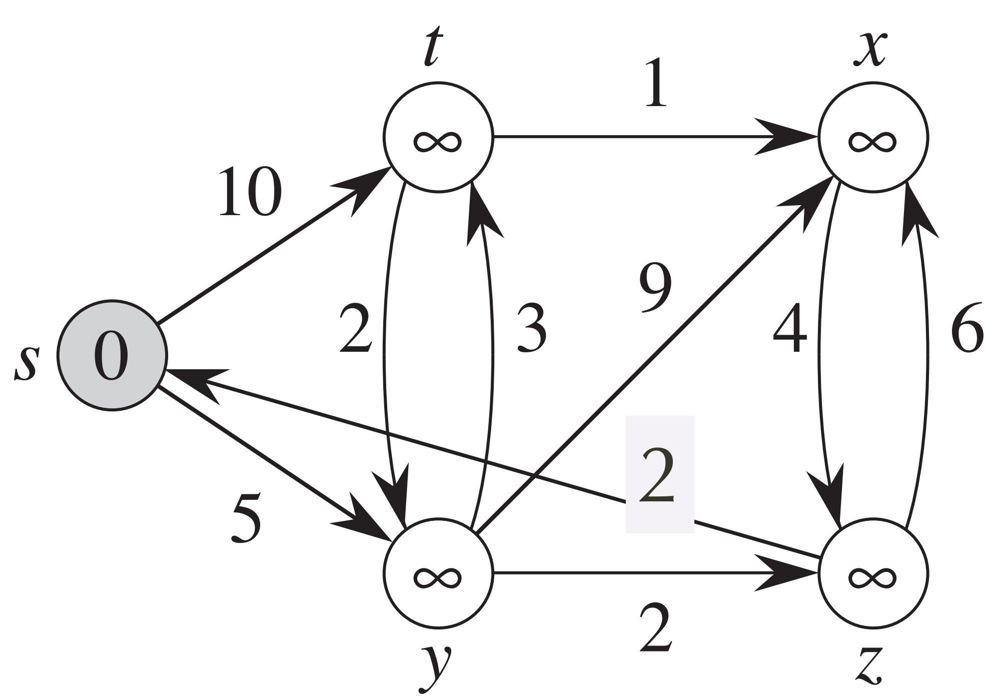
Floyd-Warshall algorithm



$$
\begin{array}{c|ccccc}
 & s & t & x & y & z \\
\hline
s & & & & & \\
t & & & & & \\
x & & & & & \\
y & & & & & \\
z & & & & &
\end{array}
\quad
\begin{array}{c}
s \\ t \\ x \\ y \\ z
\end{array}
\begin{bmatrix}
0 & 8 & 9 & 5 & 7 \\
6 & 0 & 1 & 2 & 4 \\
6 & 14 & 0 & 11 & 4 \\
4 & 3 & 4 & 0 & 2 \\
2 & 10 & 6 & 7 & 0
\end{bmatrix}
$$

The shortest path from u to v that may passes ***s, t, x, y***

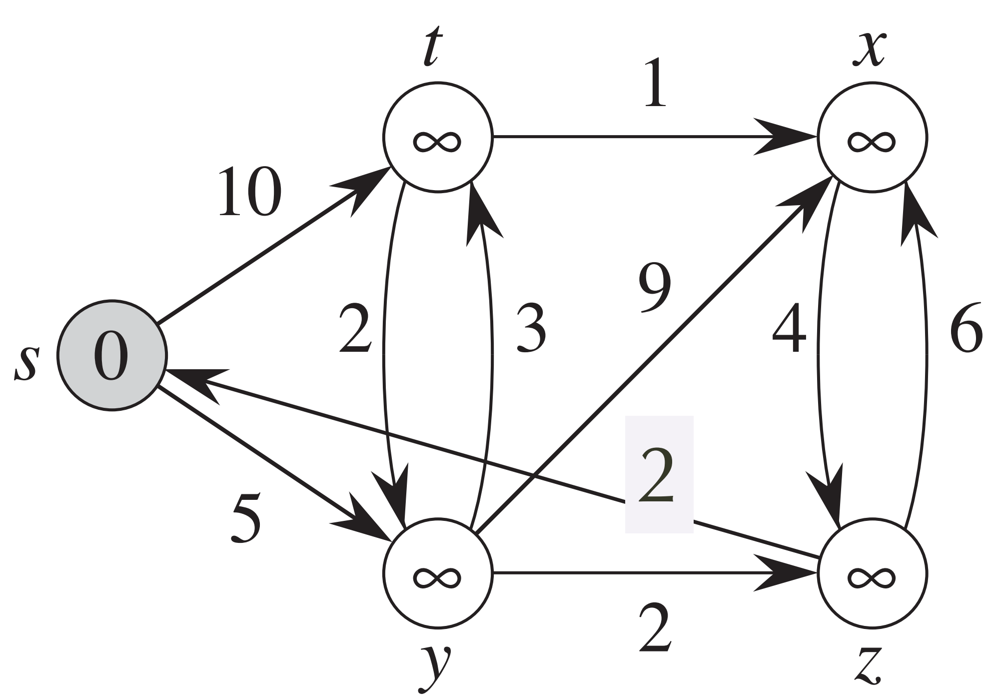
Floyd-Warshall algorithm



$$\begin{array}{c|ccccc} & s & t & x & y & z \\ \hline s & \\ t & \\ x & \\ y & \\ z & \end{array} \quad \begin{bmatrix} 0 & 8 & 9 & 5 & 7 \\ 6 & 0 & 1 & 2 & 4 \\ 6 & 14 & 0 & 11 & 4 \\ 4 & 3 & 4 & 0 & 2 \\ 2 & 10 & 6 & 7 & 0 \end{bmatrix}$$

The shortest path from u to v that may passes ***s, t, x, y***

Floyd-Warshall algorithm



$$
\begin{array}{c}
 \\ s \\ t \\ x \\ y \\ z
\end{array}
\begin{array}{c}
\begin{array}{ccccc} s & t & x & y & z \end{array} \\
\begin{bmatrix}
0 & \mathbf{8} & \mathbf{9} & 5 & \mathbf{7} \\
\mathbf{6} & 0 & 1 & 2 & \mathbf{4} \\
\mathbf{6} & \mathbf{14} & 0 & \mathbf{11} & 4 \\
\mathbf{4} & 3 & \mathbf{4} & 0 & 2 \\
2 & \mathbf{10} & 6 & \mathbf{7} & 0
\end{bmatrix}
\end{array}
$$

The shortest path from u to v that may passes ***s, t, x, y, z***