An appendix for the paper "On Bisimulation Theory in Linear Higher-Order π-Calculus"

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Approximating local bisimilarity

In this section, we define a descending chain of "bisimulation" equivalence relations (indexed by ordinal k) to approximate the local linear variant bisimilarity. We include this as an extra credit on bisimulation theory to make it more complete, and moreover, it can serve as a basis for further work such as a logical characterization.

Below is the definition of the function $\mathcal{F}$ and a family of relations $\approx^k$ ($k \leq \omega$ is an ordinal). We use $k, l..., \lambda, \kappa...$ for ordinals, I, J for index sets, and ω is the first transfinite ordinal.

Definition 1. *Define the function $\mathcal{F} : Pr_0^2 \rightarrow Pr_0^2$ as below: $P\ \mathcal{F}(\mathcal{R})\ Q$ if $\mathcal{F}(\mathcal{R})$ is closed under substitution of names and the following properties hold:*

1. *If $P \xrightarrow{\tau} P'$, then $Q \Longrightarrow Q'$ for some Q', and $P'\ \mathcal{R}\ Q'$;*
2. *If $P \xrightarrow{a(x)} P'$, then $Q \stackrel{a(x)}{\Longrightarrow} Q'$ for some Q', and $P'\ \mathcal{R}\ Q'$;*
3. *If $P \xrightarrow{\overline{a}x} P'$, then $Q \stackrel{\overline{a}x}{\Longrightarrow} Q'$ for some Q', and $P'\ \mathcal{R}\ Q'$;*
4. *If $P \xrightarrow{\overline{a}(x)} P'$, then $Q \stackrel{\overline{a}(x)}{\Longrightarrow} Q'$ for some Q', and for all (closed) processes O_1 and O_2 such that $O_1\ \mathcal{R}\ O_2$, $(x)(O_1|P')\ \mathcal{R}\ (x)(O_2|Q')$.*
5. *If $P \xrightarrow{a(\mathbf{c})} P'$, where c is a fresh name, then $Q \stackrel{a(\mathbf{c})}{\Longrightarrow} Q'$ for some Q', and $P'\ \mathcal{R}\ Q'$;*
6. *If $P \xrightarrow{(\widetilde{x})\overline{a}A} P'$, then $Q \stackrel{(\widetilde{y})\overline{a}B}{\Longrightarrow} Q'$ for some $\widetilde{y}, B, Q'$. And for a process $E[X]$ of the form $\overline{c}.(X+d)$, where c, d are fresh names, it holds that $(\widetilde{x})(E[A]|P')\ \mathcal{R}\ (\widetilde{y})(E[B]|Q')$.*

And vice versa.

Now the hierarchy $\approx^k$ ($k < \omega$ is an ordinal) can be defined as (λ is a transfinite ordinal):

$$\begin{aligned} \approx^0 &= Pr_0^2 \\ \approx^{k+1} &= \mathcal{F}(\approx^k) \\ \approx^\lambda &= \bigcap_{k<\lambda} \approx^k \end{aligned}$$

The relation $\approx^k$ can be extended to open processes in the usual fashion.

Based on the definition above, we have the following two important propositions.

Proposition 1 ($\mathcal{F}$ properties). *Suppose $2^{Pr_0^2}$ is a complete lattice with the order of set inclusion on it. Then the following properties hold:*

1. *$\mathcal{F}$ is monotone. That is If $k < k'$, then $\approx^{k'} \subseteq \approx^k$.*
2. *$\mathcal{R}$ is a bisimulation iff $\mathcal{R} \subseteq \mathcal{F}(\mathcal{R})$;*
3. *If $\{X_i\}_{i\in I}$ is a codirected set, then $\mathcal{F}(\bigcap_{i\in I} X_i) = \bigcap_{i\in I} \mathcal{F}(X_i)$;*
4. *The greatest bisimulation $\approx_{ll}^v$ exists and it coincides with $\approx^\omega$. That is $\approx^\omega = \approx_{ll}^v$.*

Proof. The proof is of the traditional style like that in [1] [2] [3] [4]. As an example, we focus on 4 to show that $\approx^\omega = \approx_{ll}^v$. The existence of $\approx^\omega$ is not hard.

"$\supseteq$". By induction on the ordinal $k < \omega$. When $k = 0$, it is obvious that $\approx_{ll}^v \subseteq \approx^0$. Now suppose $\approx_{ll}^v \subseteq \approx^k$, we show that $\approx_{ll}^v \subseteq \approx^{k+1}$. Suppose $P \approx_{ll}^v Q$. We have the following analysis:

1. If $P \xrightarrow{\tau} P'$, then $Q \Longrightarrow Q'$ for some Q', and $P' \approx_{ll}^v Q'$;
2. If $P \xrightarrow{a(x)} P'$, then $Q \stackrel{a(x)}{\Longrightarrow} Q'$ for some Q', and $P' \approx_{ll}^v Q'$;

3. If $P \xrightarrow{\overline{a}x} P'$, then $Q \stackrel{\overline{a}x}{\Longrightarrow} Q'$ for some Q', and $P' \approx_{ll}^{v} Q'$;
4. If $P \xrightarrow{\overline{a}(x)} P'$, then $Q \stackrel{\overline{a}(x)}{\Longrightarrow} Q'$ for some Q', and for all (closed) processes O_1 and O_2 such that $O_1 \approx_{ll}^{v} O_2$, $(x)(O_1|P') \approx_{ll}^{v} (x)(O_2|Q')$.
5. If $P \xrightarrow{a(\mathbf{c})} P'$, where c is a fresh name, then $Q \stackrel{a(\mathbf{c})}{\Longrightarrow} Q'$ for some Q', and $P' \approx_{ll}^{v} Q'$;
6. If $P \xrightarrow{(\widetilde{x})\overline{a}A} P'$, then $Q \stackrel{(\widetilde{y})\overline{a}B}{\Longrightarrow} Q'$ for some $\widetilde{y}, B, Q'$. And for a process $E[X]$ of the form $\overline{c}.(X+d)$, where c, d are fresh names, it holds that $(\widetilde{x})(E[A]|P') \approx_{ll}^{v} (\widetilde{y})(E[B]|Q')$.

This suffices to show that $\approx_{ll}^{v} \subseteq \approx^{k+1}$, by induction hypothesis.

"$\supseteq$". We show that $\approx^{\omega}$ is a local linear variant bisimulation (through definition checking). Suppose $P \approx^{\omega} Q$, then for every $(k+1) < \omega$, $P \approx^{k+1} Q$. Thus we have the analysis below:

1. If $P \xrightarrow{\tau} P'$, then $Q \Longrightarrow Q'$ for some Q', and $P' \approx^{k} Q'$;
2. If $P \xrightarrow{a(x)} P'$, then $Q \stackrel{a(x)}{\Longrightarrow} Q'$ for some Q', and $P' \approx^{k} Q'$;
3. If $P \xrightarrow{\overline{a}x} P'$, then $Q \stackrel{\overline{a}x}{\Longrightarrow} Q'$ for some Q', and $P' \approx^{k} Q'$;
4. If $P \xrightarrow{\overline{a}(x)} P'$, then $Q \stackrel{\overline{a}(x)}{\Longrightarrow} Q'$ for some Q', and for all (closed) processes O_1 and O_2 such that $O_1 \approx^{k} O_2$, $(x)(O_1|P') \approx^{k} (x)(O_2|Q')$.
5. If $P \xrightarrow{a(\mathbf{c})} P'$, where c is a fresh name, then $Q \stackrel{a(\mathbf{c})}{\Longrightarrow} Q'$ for some Q', and $P' \approx^{k} Q'$;
6. If $P \xrightarrow{(\widetilde{x})\overline{a}A} P'$, then $Q \stackrel{(\widetilde{y})\overline{a}B}{\Longrightarrow} Q'$ for some $\widetilde{y}, B, Q'$. And for a process $E[X]$ of the form $\overline{c}.(X+d)$, where c, d are fresh names, it holds that $(\widetilde{x})(E[A]|P') \approx^{k} (\widetilde{y})(E[B]|Q')$.

In any case, P can be matched by Q, and it holds for every $k+1$. By this and a standard argument on ordinals, we conclude that $\approx^{\omega}$ is a local linear variant bisimulation.

Another is on the congruence property of $\approx^{k}$.

Proposition 2 (Congruence of $\approx^{k}$). *The relation $\approx^{k}$ ($k \leq \omega$) is a congruence with respect to all the operators in the calculus except the choice operator. That is, suppose $P_i \approx^{k} Q_i$ ($i = 1, 2$) and $P \approx^{k} Q$, then*

$$\begin{array}{l}
\tau.P \approx^{k} \tau.Q \\
c(x).P \approx^{k} c(x).Q, \quad \overline{c}x.P \approx^{k} \overline{c}x.Q \\
c(X).P \approx^{k} c(X).Q, \quad \overline{c}P_1.P_2 \approx^{k} \overline{c}Q_1.Q_2 \\
(x)P \approx^{k} (x)Q \\
P_1|P_2 \approx^{k} Q_1|Q_2
\end{array}$$

Proof. Routine check using similar approach to that in the proof of the congruence of $\approx_{ll}^{v}$.

References

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