Appendix

A The Complete Model in κ for DNA Addition

In this appendix, we present the complete κ model for the DNA addition algorithm from [11].

A.1 Activation

- On $A_i (i = 1, 2, \cdots, n-1)$

$$
\begin{array}{rcl}
A_i^{00}(c_0 s_0^x + \overline{h_0}) & \longrightarrow & A_i^{00}(c_0 s_0^x + h_0) \\
A_i^{01}(c_0 s_1^x + \overline{h_0}) & \longrightarrow & A_i^{01}(c_0 s_1^x + h_0) \\
A_i^{02}(c_1 s_0^x + \overline{h_1}) & \longrightarrow & A_i^{02}(c_1 s_0^x + h_1) \\
A_i^{03}(c_1 s_1^x + \overline{h_1}) & \longrightarrow & A_i^{03}(c_1 s_1^x + h_1) \\
A_i^{10}(c_0 s_0^x + \overline{h_1}) & \longrightarrow & A_i^{10}(c_0 s_0^x + h_1) \\
A_i^{11}(c_0 s_1^x + \overline{h_1}) & \longrightarrow & A_i^{11}(c_0 s_1^x + h_1) \\
A_i^{12}(c_1 s_0^x + \overline{h_2}) & \longrightarrow & A_i^{12}(c_1 s_0^x + h_2) \\
A_i^{13}(c_1 s_1^x + \overline{h_2}) & \longrightarrow & A_i^{13}(c_1 s_1^x + h_2)
\end{array}
$$

- On $B_i (i = 1, 2, \cdots, n-1)$

$$
\begin{array}{rcl}
B_i^{00}(h_0^x + \overline{c_0 s_0}) & \longrightarrow & B_i^{00}(h_0^x + c_0 s_0) \\
B_i^{01}(h_1^x + \overline{c_0 s_1}) & \longrightarrow & B_i^{01}(h_1^x + c_0 s_1) \\
B_i^{02}(h_2^x + \overline{c_1 s_0}) & \longrightarrow & B_i^{02}(h_2^x + c_1 s_0) \\
B_i^{10}(h_0^x + \overline{c_0 s_1}) & \longrightarrow & B_i^{10}(h_0^x + c_0 s_1) \\
B_i^{11}(h_1^x + \overline{c_1 s_0}) & \longrightarrow & B_i^{11}(h_1^x + c_1 s_0) \\
B_i^{12}(h_2^x + \overline{c_1 s_1}) & \longrightarrow & B_i^{12}(h_2^x + c_1 s_1)
\end{array}
$$

A.2 Connection$(i = 1, 2, \cdots, n-1)$

- On h_0

$$
\begin{array}{rcl}
A_i^{00}(c_0 s_0^x + h_0), B_i^{00}(h_0 + \overline{c_0 s_0}) & \longrightarrow & (y)(A_i^{00}(c_0 s_0^x + h_0^y), B_i^{00}(h_0^y + \overline{c_0 s_0})) \\
A_i^{00}(c_0 s_0^x + h_0), B_i^{10}(h_0 + \overline{c_0 s_1}) & \longrightarrow & (y)(A_i^{00}(c_0 s_0^x + h_0^y), B_i^{10}(h_0^y + \overline{c_0 s_1})) \\
A_i^{01}(c_0 s_1^x + h_0), B_i^{00}(h_0 + \overline{c_0 s_0}) & \longrightarrow & (y)(A_i^{01}(c_0 s_1^x + h_0^y), B_i^{00}(h_0^y + \overline{c_0 s_0})) \\
A_i^{01}(c_0 s_1^x + h_0), B_i^{10}(h_0 + \overline{c_0 s_1}) & \longrightarrow & (y)(A_i^{01}(c_0 s_1^x + h_0^y), B_i^{10}(h_0^y + \overline{c_0 s_1}))
\end{array}
$$

- On h_1

$$
\begin{array}{rcl}
A_i^{02}(c_1 s_0^x + h_1), B_i^{01}(h_1 + \overline{c_0 s_1}) & \longrightarrow & (y)(A_i^{02}(c_1 s_0^x + h_1^y), B_i^{01}(h_1^y + \overline{c_0 s_1})) \\
A_i^{02}(c_1 s_0^x + h_1), B_i^{11}(h_1 + \overline{c_1 s_0}) & \longrightarrow & (y)(A_i^{02}(c_1 s_0^x + h_1^y), B_i^{11}(h_1^y + \overline{c_1 s_0})) \\
A_i^{03}(c_1 s_1^x + h_1), B_i^{01}(h_1 + \overline{c_0 s_1}) & \longrightarrow & (y)(A_i^{03}(c_1 s_1^x + h_1^y), B_i^{01}(h_1^y + \overline{c_0 s_1})) \\
A_i^{03}(c_1 s_1^x + h_1), B_i^{11}(h_1 + \overline{c_1 s_0}) & \longrightarrow & (y)(A_i^{03}(c_1 s_1^x + h_1^y), B_i^{11}(h_1^y + \overline{c_1 s_0})) \\
A_i^{10}(c_0 s_0^x + h_1), B_i^{01}(h_1 + \overline{c_0 s_1}) & \longrightarrow & (y)(A_i^{10}(c_0 s_0^x + h_1^y), B_i^{01}(h_1^y + \overline{c_0 s_1}))
\end{array}
$$

$$
\begin{array}{rcl}
A_i^{10}(c_0 s_0^x + h_1), B_i^{11}(h_1 + \overline{c_1 s_0}) & \longrightarrow & (y)(A_i^{10}(c_0 s_0^x + h_1^y), B_i^{11}(h_1^y + \overline{c_1 s_0})) \\
A_i^{11}(c_0 s_1^x + h_1), B_i^{01}(h_1 + \overline{c_0 s_1}) & \longrightarrow & (y)(A_i^{11}(c_0 s_1^x + h_1^y), B_i^{01}(h_1^y + \overline{c_0 s_1})) \\
A_i^{11}(c_0 s_1^x + h_1), B_i^{11}(h_1 + \overline{c_1 s_0}) & \longrightarrow & (y)(A_i^{11}(c_0 s_1^x + h_1^y), B_i^{01}(h_1^y + \overline{c_1 s_0}))
\end{array}
$$

- On h_2

$$
\begin{array}{rcl}
A_i^{12}(c_1 s_0^x + h_2), B_i^{02}(h_2 + \overline{c_1 s_0}) & \longrightarrow & (y)(A_i^{12}(c_1 s_0^x + h_2^y), B_i^{02}(h_2^y + \overline{c_1 s_0})) \\
A_i^{12}(c_1 s_0^x + h_2), B_i^{12}(h_2 + \overline{c_1 s_1}) & \longrightarrow & (y)(A_i^{12}(c_1 s_0^x + h_2^y), B_i^{12}(h_2^y + \overline{c_1 s_1})) \\
A_i^{13}(c_1 s_1^x + h_2), B_i^{02}(h_2 + \overline{c_1 s_0}) & \longrightarrow & (y)(A_i^{13}(c_1 s_1^x + h_2^y), B_i^{02}(h_2^y + \overline{c_1 s_0})) \\
A_i^{13}(c_1 s_1^x + h_2), B_i^{12}(h_2 + \overline{c_1 s_1}) & \longrightarrow & (y)(A_i^{13}(c_1 s_1^x + h_2^y), B_i^{12}(h_2^y + \overline{c_1 s_1}))
\end{array}
$$

- On $c_0 s_0$

$$
\begin{array}{rcl}
B_{i-1}^{00}(h_0^x + c_0 s_0), A_i^{00}(c_0 s_0 + \overline{h_0}) & \longrightarrow & (y)(B_{i-1}^{00}(h_0^x + c_0 s_0^y), A_i^{00}(c_0 s_0^y + \overline{h_0})) \\
B_{i-1}^{00}(h_0^x + c_0 s_0), A_i^{10}(c_0 s_0 + \overline{h_1}) & \longrightarrow & (y)(B_{i-1}^{00}(h_0^x + c_0 s_0^y), A_i^{10}(c_0 s_0^y + \overline{h_1}))
\end{array}
$$

- On $c_0 s_1$

$$
\begin{array}{rcl}
B_{i-1}^{01}(h_1^x + c_0 s_1), A_i^{01}(c_0 s_1 + \overline{h_0}) & \longrightarrow & (y)(B_{i-1}^{01}(h_1^x + c_0 s_1^y), A_i^{01}(c_0 s_1^y + \overline{h_0})) \\
B_{i-1}^{01}(h_1^x + c_0 s_1), A_i^{11}(c_0 s_1 + \overline{h_1}) & \longrightarrow & (y)(B_{i-1}^{01}(h_1^x + c_0 s_1^y), A_i^{11}(c_0 s_1^y + \overline{h_1})) \\
B_{i-1}^{10}(h_0^x + c_0 s_1), A_i^{01}(c_0 s_1 + \overline{h_0}) & \longrightarrow & (y)(B_{i-1}^{10}(h_0^x + c_0 s_1^y), A_i^{01}(c_0 s_1^y + \overline{h_0})) \\
B_{i-1}^{10}(h_0^x + c_0 s_1), A_i^{11}(c_0 s_1 + \overline{h_1}) & \longrightarrow & (y)(B_{i-1}^{10}(h_0^x + c_0 s_1^y), A_i^{11}(c_0 s_1^y + \overline{h_1}))
\end{array}
$$

- On $c_1 s_0$

$$
\begin{array}{rcl}
B_{i-1}^{02}(h_2^x + c_1 s_0), A_i^{02}(c_1 s_0 + \overline{h_1}) & \longrightarrow & (y)(B_{i-1}^{02}(h_2^x + c_1 s_0^y), A_i^{02}(c_1 s_0^y + \overline{h_1})) \\
B_{i-1}^{02}(h_2^x + c_1 s_0), A_i^{12}(c_1 s_0 + \overline{h_2}) & \longrightarrow & (y)(B_{i-1}^{02}(h_2^x + c_1 s_0^y), A_i^{12}(c_1 s_0^y + \overline{h_2})) \\
B_{i-1}^{11}(h_1^x + c_1 s_0), A_i^{02}(c_1 s_0 + \overline{h_1}) & \longrightarrow & (y)(B_{i-1}^{11}(h_1^x + c_1 s_0^y), A_i^{02}(c_1 s_0^y + \overline{h_1})) \\
B_{i-1}^{11}(h_1^x + c_1 s_0), A_i^{12}(c_1 s_0 + \overline{h_2}) & \longrightarrow & (y)(B_{i-1}^{11}(h_1^x + c_1 s_0^y), A_i^{12}(c_1 s_0^y + \overline{h_2}))
\end{array}
$$

- On $c_1 s_1$

$$
\begin{array}{rcl}
B_{i-1}^{12}(h_2^x + c_1 s_1), A_i^{03}(c_1 s_1 + \overline{h_1}) & \longrightarrow & (y)(B_{i-1}^{12}(h_2^x + c_1 s_1^y), A_i^{03}(c_1 s_1^y + \overline{h_1})) \\
B_{i-1}^{12}(h_2^x + c_1 s_1), A_i^{13}(c_1 s_1 + \overline{h_2}) & \longrightarrow & (y)(B_{i-1}^{12}(h_2^x + c_1 s_1^y), A_i^{13}(c_1 s_1^y + \overline{h_2}))
\end{array}
$$

A.3 Beginning and Ending

- Beginning

$$
\begin{array}{rcl}
A_0^0(\overline{PrimerL} + h_0^x) & \longrightarrow & A_0^0(PrimerL + h_0^x) \\
A_0^1(\overline{PrimerL} + h_1^x) & \longrightarrow & A_0^1(PrimerL + h_1^x) \\
A_0^0(\overline{PrimerL} + h_0), B_0^{00}(h_0 + \overline{c_0 s_0}) & \longrightarrow & (y)(A_0^0(\overline{PrimerL} + h_0^y), B_0^{00}(h_0^y + \overline{c_0 s_0})) \\
A_0^0(\overline{PrimerL} + h_0), B_0^{10}(h_0 + \overline{c_0 s_1}) & \longrightarrow & (y)(A_0^0(\overline{PrimerL} + h_0^y), B_0^{10}(h_0^y + \overline{c_0 s_1})) \\
A_0^1(\overline{PrimerL} + h_1), B_0^{01}(h_1 + \overline{c_0 s_1}) & \longrightarrow & (y)(A_0^1(\overline{PrimerL} + h_1^y), B_0^{01}(h_1^y + \overline{c_0 s_1})) \\
A_0^1(\overline{PrimerL} + h_1), B_0^{11}(h_1 + \overline{c_1 s_0}) & \longrightarrow & (y)(A_0^1(\overline{PrimerL} + h_1^y), B_0^{11}(h_1^y + \overline{c_1 s_0}))
\end{array}
$$

- Ending$(p, q = 0, 1)$

$$
\begin{aligned}
BEnd_n^{pq}(c_p s_q^x + \overline{PrimerR}) &\longrightarrow BEnd_n^{pq}(c_p s_q^x + PrimerR)) \\
BEnd_n^{00}(c_0 s_0 + \overline{PrimerR}), B_{n-1}^{00}(h_0^x + c_0 s_0) &\longrightarrow \\
(y)(BEnd_n^{00}(c_0 s_0^y + \overline{PrimerR}), B_{n-1}^{00}(h_0^x + c_0 s_0^y)) & \\
BEnd_n^{01}(c_0 s_1 + \overline{PrimerR}), B_{n-1}^{01}(h_1^x + c_0 s_1) &\longrightarrow \\
(y)(BEnd_n^{01}(c_0 s_1^y + \overline{PrimerR}), B_{n-1}^{01}(h_1^x + c_0 s_1^y)) & \\
BEnd_n^{01}(c_0 s_1 + \overline{PrimerR}), B_{n-1}^{10}(h_0^x + c_0 s_1) &\longrightarrow \\
(y)(BEnd_n^{01}(c_0 s_1^y + \overline{PrimerR}), B_{n-1}^{10}(h_0^x + c_0 s_1^y)) & \\
BEnd_n^{10}(c_1 s_0 + \overline{PrimerR}), B_{n-1}^{02}(h_2^x + c_1 s_0) &\longrightarrow \\
(y)(BEnd_n^{10}(c_1 s_0^y + \overline{PrimerR}), B_{n-1}^{02}(h_2^x + c_1 s_0^y)) & \\
BEnd_n^{10}(c_1 s_0 + \overline{PrimerR}), B_{n-1}^{11}(h_1^x + c_1 s_0) &\longrightarrow \\
(y)(BEnd_n^{10}(c_1 s_0^y + \overline{PrimerR}), B_{n-1}^{11}(h_1^x + c_1 s_0^y)) & \\
BEnd_n^{11}(c_1 s_1 + \overline{PrimerR}), B_{n-1}^{12}(h_2^x + c_1 s_1) &\longrightarrow \\
(y)(BEnd_n^{11}(c_1 s_1^y + \overline{PrimerR}), B_{n-1}^{12}(h_2^x + c_1 s_1^y)) &
\end{aligned}
$$